


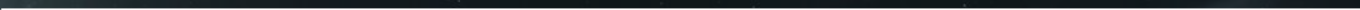
Celestial Mechanics: Application of Kepler's Laws and Spherical Trigonometry

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01



Introduction



Table of Contents

01

Kepler's Laws

Newton's Universal
Law of Gravitation



Kepler's Three
Laws of Planetary
Motion

02

Trigonometry

Law of Sines
and Cosines



Trigonometry
of Spherical
Geometry

03

Polar Ellipses

Rectangular
Ellipses



Polar
Representation of
Shifted Ellipse

04

Connection

Processes (01),
(02), (03)



Right Ascension
and Declination for
Celestial Objects



02 ✨

Kepler's Laws



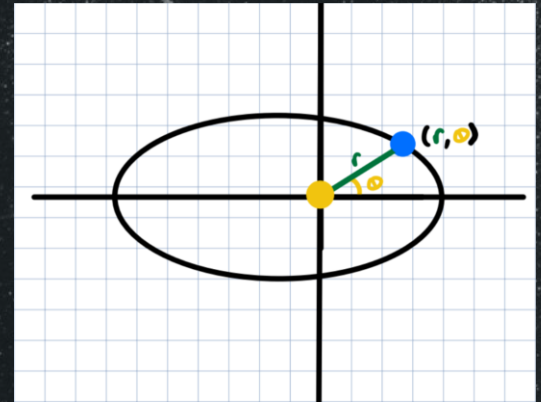


Kepler's First Law

Planets orbit in elliptical paths
with their star as a focus.

Kepler's First Law

- Planets orbit around their host star
- These orbits are elliptical in nature
- The star occupies one focus of this ellipse



The equation for the planet's orbit

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta}$$

r is the distance from the planet to the star, a is the length of the orbit's semi major axis and e is the eccentricity of the orbit.



Kepler's Second Law

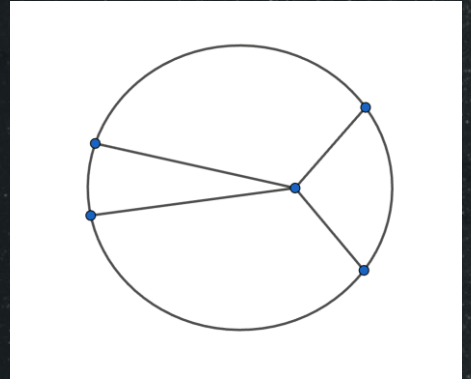
The radius from the star to the planet sweeps out equal areas in equal amounts of time.

Kepler's Second Law

As a planet moves around its star, the radius between the bodies sweeps out an area which remains constant per unit time.

The change in area is given by:

$$dA = \frac{1}{2} r^2 \left(\frac{h}{r^2} \right) dt = \frac{h}{2} dt$$




Where h is a constant such that $h^2 = GMa(1 - e^2)$ and r is the distance between the planet and its star.



Kepler's Third Law

The square of the period of a planet's orbit is proportional to the cube of its semi-major axis.



Kepler's Third Law

- The area of an ellipse is πab
- a is the length of the semi-major axis and b is the length of the semi-minor axis.
- Let T represent the period of the planet's orbit

The area can be calculated by integrating dA from 0 to T . From Kepler's 2nd Law, $dA = (h/2)dt$. After integrating and solving for T^2 , the result is:

$$T^2 = \frac{4\pi^2 a^2 b^2}{h^2}$$

Since $h^2 = GMa(1 - e^2)$, the final result is:

$$T^2 = \frac{4\pi^2}{GM} a^3$$

Conservation of Energy

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2$$

$$r = \frac{a(1 - e^2)}{1 + e\cos\theta}$$

$$E = -\frac{GMm}{2a}$$

Energy is conserved.

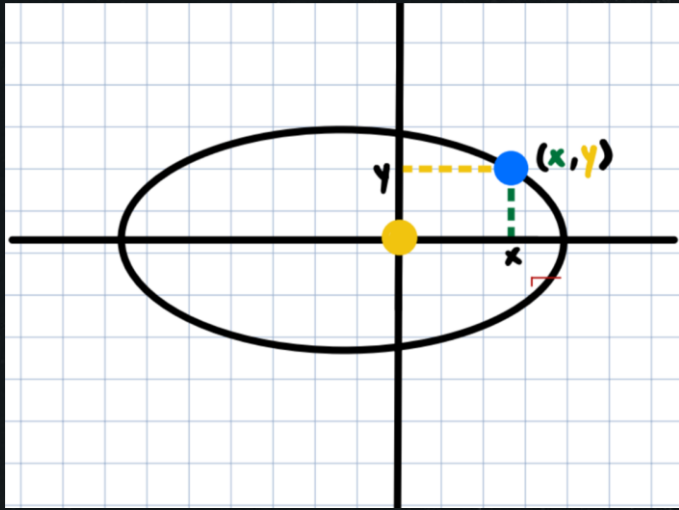
03



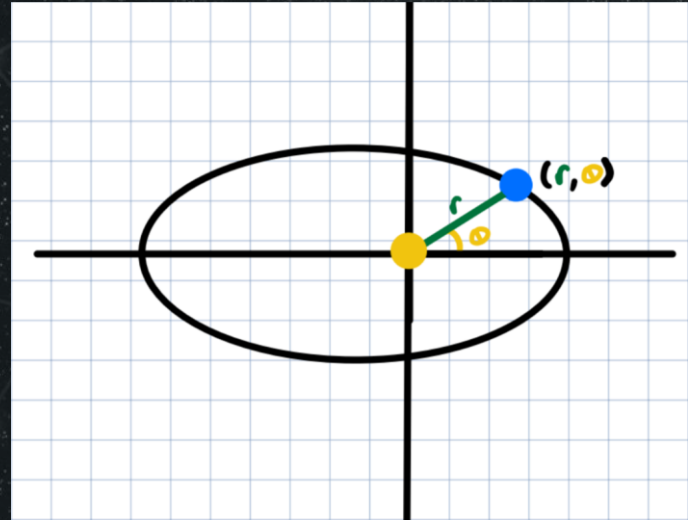
Ellipse Geometry



Ellipse Geometry



$$\frac{(x+c)^2}{a^2} + \frac{y^2}{b^2} = 1$$



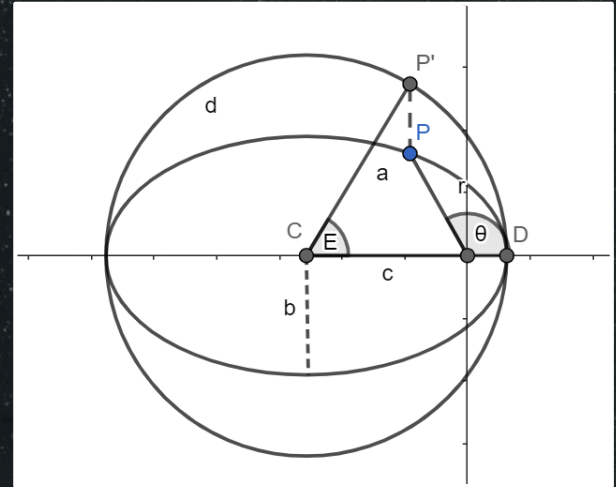
$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

Connecting E to θ

$$\cos E = \frac{\cos\theta + e}{1 + e\cos\theta}$$

$$r\sin\theta = b\sin E$$

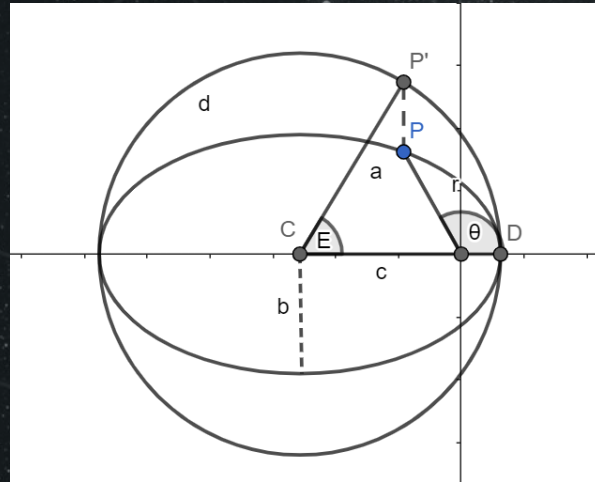
$$\tan\left(\frac{E}{2}\right) = \sqrt{\left(\frac{1+e}{1-e}\right)} \tan\left(\frac{\theta}{2}\right)$$



Showing that $\frac{dM}{dt}$ is constant

$$M = E - e \sin E$$

$$\frac{dM}{dt} = \frac{h}{ab} = \frac{2\pi}{T}$$



Now we've connected time to θ

04 ✨

Spherical Trigonometry



Spherical Trigonometry

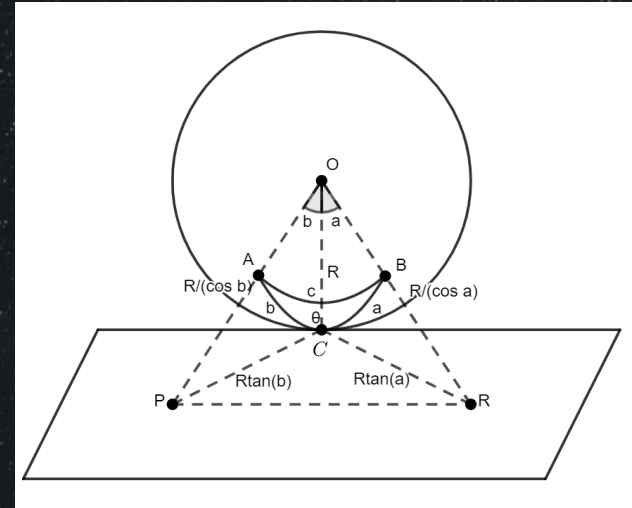
- Geometry on a spherical surface has different rules
- Shortest path between two points lies on a "Great Circle"
- Spherical triangles are formed by these arcs

Spherical Law of Sines

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

Spherical Law of Cosines

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$



05



Celestial Sphere



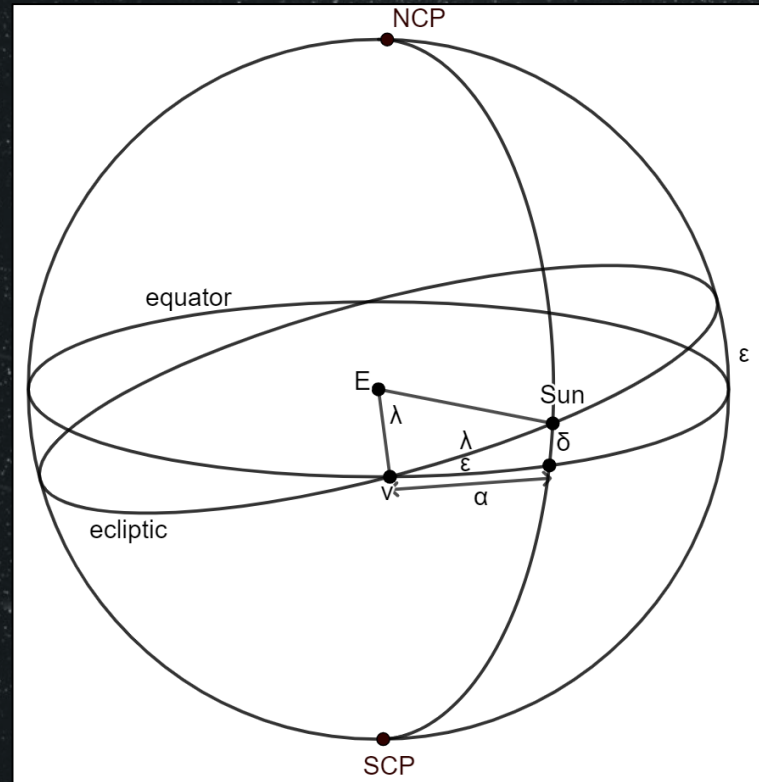
A Shift in Reference Frame.....

Principles

- An apparent sphere formed by the sky (and its other half below the horizon)
- Rotates East to West
- Longitude, Latitude \rightarrow Right Ascension, Declination

Assumptions

- Arbitrarily large distances



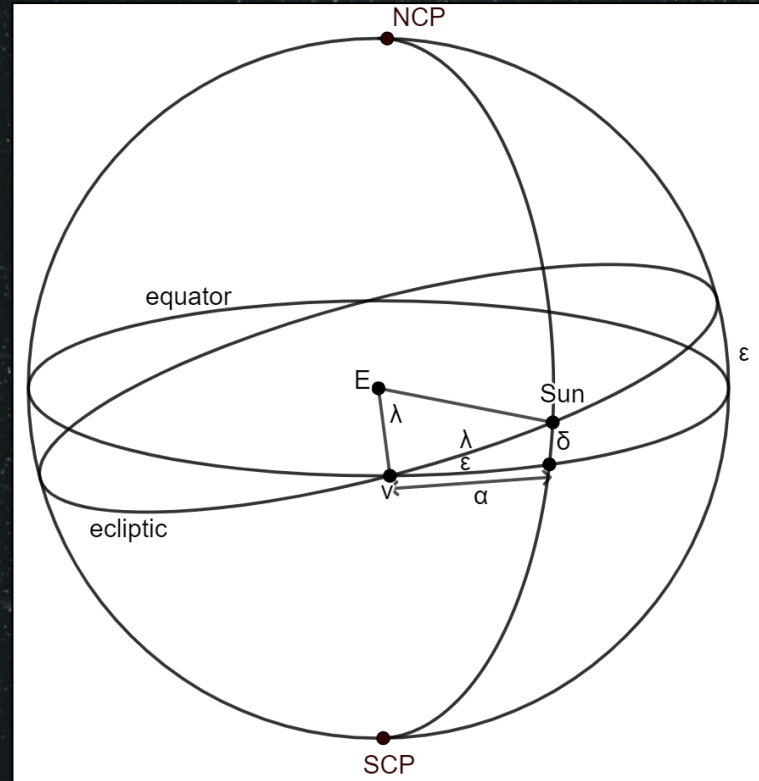
Points of Reference

Equatorial Coordinates

- Celestial Equator
- Vernal Equinox
- Celestial Poles

Ecliptic Coordinates

- Ecliptic
- Vernal Equinox
- Ecliptic Poles



06

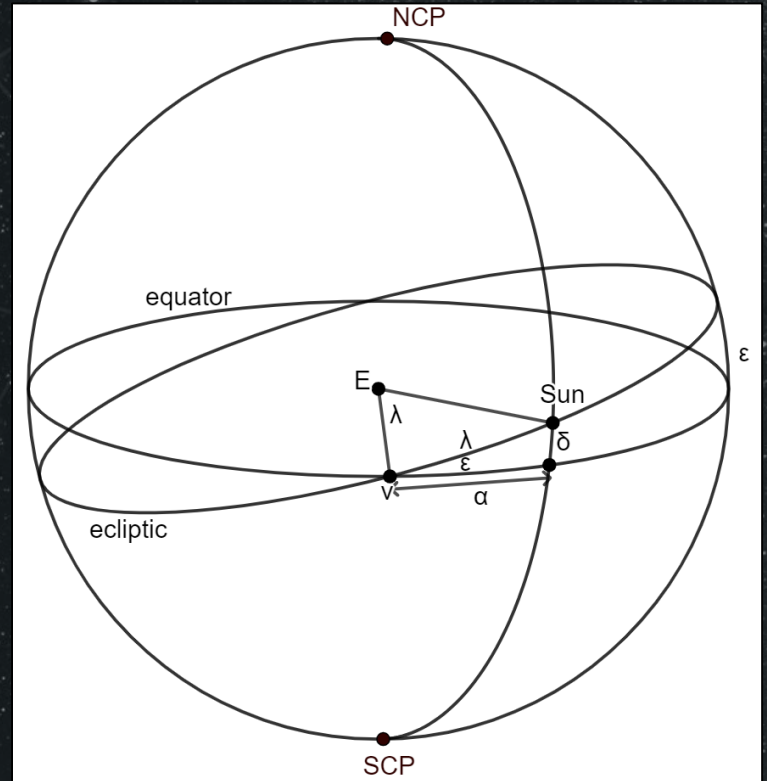


Rise & Set Times



☀ Sunrise and Sunset

- declination determines the path of the sun in the sky
- one hemisphere visible from a given point on the Earth
 - time the sun spends in that hemisphere is the time the sun is in the sky
- highest point is noon
 - subtract off for sunrise
 - add on for sunset



Saturn Rise and Saturn Set

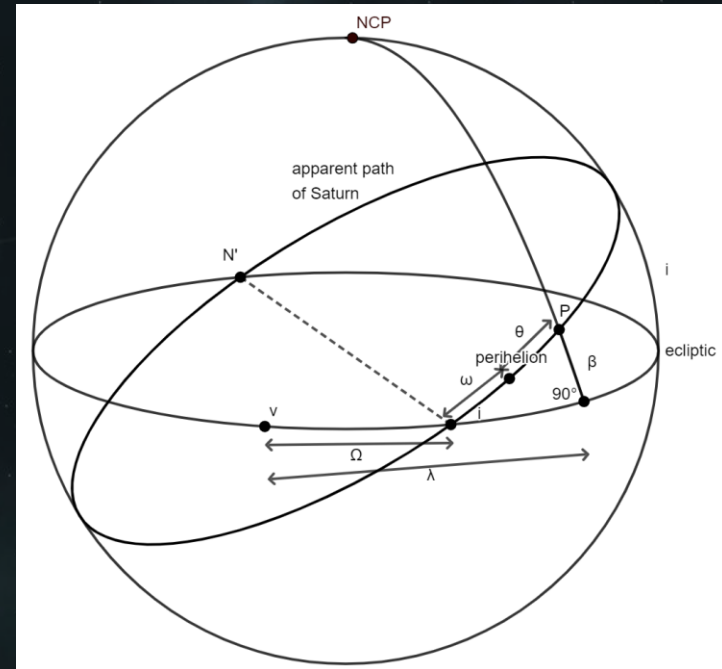
- Given θ for Saturn, we can calculate "ecliptic" coordinates



- Given ecliptic coordinates, we can calculate Right Ascension and Declination



- Given RA and Decl., we can calculate time spent in the sky.



08 ✨

Machine Learning – Error Analysis



Generating Data: Kepler's Laws



Saturn Constants							Last Perihelion			
a	e	i	T (days)	P	Q	ω	Date			
113.665	339.392	2.485	0.0565	10755.7	23.5		July 26, 2003			
dM/dt										
0.0005841726068										
End Date	t	M	E	θ	β	λ	δ	α	α (formatted)	δ (formatted)
01/31/2020	603.8	3.524313337	3.504268424	201.9603639	-0.05863013754	295.0160926	-21.24030131	296.9808229	19h 47m 55.5s	-22° 45'34.92"
02/29/2020	6062	3.541254343	3.520361928	202.9321565	-0.1007428136	295.9869733	-21.10295526	298.0124981	19h 52m 3s	-22° 53'49.36"
03/31/2020	6093	3.559363693	3.53575102	203.9709312	-0.1457258698	297.024776	-20.94925723	299.0131247	19h 56m 27.15s	-21° 30'46.7"
04/30/2020	6123	3.576888872	3.55423082	204.9761583	-0.1892108143	298.0290663	-20.79381502	300.1760076	20h 0m 42.24s	-21° 12'22.27"
05/31/2020	6154	3.594998222	3.571452234	206.0148515	-0.2340822446	299.066797	-20.62633884	301.2719292	20h 5m 5.26s	-21° 22'25.18"
06/30/2020	6184	3.612523401	3.588124442	207.0199972	-0.2774314709	300.0710176	-20.45770597	302.3301151	20h 9m 19.23s	-21° 32'32.26"
07/31/2020	6215	3.630632751	3.60549111	208.0096039	-0.32134175	300.00886	-20.27675224	303.4210427	20h 13m 41.05s	-21° 43'23.69"
08/31/2020	6246	3.648742102	3.62281111	209.0091547	-0.366729148	300.00886	-20.0937063	304.5093323	20h 18m 2.24s	-21° 54'39.35"
09/30/2020	6276	3.66626728	3.63910379	210.0091547	-0.41271111	300.00886	-19.9147741	305.599462	20h 22m 14.39s	-20° 5'55.98"
10/31/2020	6307	3.684376631	3.65655066	211.1406831	-0.454109759	304.1679626	-19.7001719	306.6428696	20h 26m 34.29s	-20° 17'58.39"
11/30/2020	6337	3.701901809	3.67325803	212.1455676	-0.4968762776	305.1920082	-19.50008957	307.6881875	20h 30m 45.16s	-20° 29'59.68"
12/31/2020	6368	3.720011116	3.690530509	213.1839371	-0.5409058174	306.2294865	-19.2867762	308.765541	20h 35m 3.73s	-20° 42'47.61"
01/31/2021	6399	3.738120511	3.707811383	214.2222555	-0.5847558655	307.2669286	-19.06717495	309.839969	20h 39m 21.6s	-20° 55'58.17"
02/28/2021	6427	3.754477344	3.723427336	215.1600469	-0.6241961135	308.2039424	-18.86349986	310.8079502	20h 43m 13.91s	-19° 8'11.4"
03/31/2021	6458	3.772586695	3.740724873	216.1982657	-0.6676648527	309.2413168	-18.63219527	311.8767875	20h 47m 30.43s	-19° 22'4.1"
04/30/2021	6488	3.790111873	3.757473099	217.2029426	-0.709521031	310.2451939	-18.4026263	312.9082918	20h 51m 37.99s	-19° 35'50.53"
05/31/2021	6519	3.808221224	3.7747888	218.2410554	-0.7525412631	311.2824994	-18.15960076	313.9712049	20h 55m 53.09s	-19° 50'25.44"
06/30/2021	6549	3.825746402	3.791555061	219.2456284	-0.7939369193	312.2863106	-17.9188773	314.9969317	20h 59m 59.26s	-18° 45'2.04"
07/31/2021	6580	3.843855753	3.808889868	220.2836327	-0.8364540757	313.3235487	-17.66451245	316.0538375	21h 4m 12.92s	-18° 20'7.76"
08/31/2021	6611	3.861965104	3.826234741	221.321581	-0.8786948745	314.3607534	-17.40454089	317.1076695	21h 8m 25.84s	-18° 35'43.65"
09/30/2021	6641	3.879490282	3.843029911	222.3259933	-0.9192969184	315.3644683	-17.14771599	318.1245724	21h 12m 29.9s	-18° 51'18.22"
10/31/2021	6672	3.897599633	3.860395286	223.3638302	-0.9609536546	316.4016085	-16.87701926	319.1723357	21h 16m 41.36s	-17° 7'22.73"
11/30/2021	6702	3.915124811	3.877210738	224.3681341	-1.000964864	317.4052622	-16.61001468	320.1833619	21h 20m 44.01s	-17° 23'23.95"
12/31/2021	6733	3.933234162	3.894597523	225.4058583	-1.041984809	318.4423402	-16.32900694	321.2250568	21h 24m 54.01s	-17° 40'15.58"
01/31/2022	6764	3.951343513	3.911995533	226.443575	-1.082661201	319.4793875	-16.0429201	322.2636678	21h 29m 3.28s	-17° 57'25.49"

**INPUTTED
CONSTANTS**

For each planet,
constants were found
and inputted

**PROGRAMMED
CALCULATIONS**

Used mathematical
equations derived
previously to generate
hundreds of data values
from many times

SATURN

Generating Data: Kepler's Laws

Orbit Constants										Planet Constants										
G	a	i	e	T (days)	ρ	ρ	ρ	ρ	ρ	ρ	ρ	ρ	ρ	ρ	ρ	ρ	ρ	ρ	ρ	ρ
74.066	96.560	0.773	0.0457	39.689	23.5															
Last Perihelion January 15, 2023										Last Perihelion January 23, 2023										
dM = 0.00204407109										dM = 0.001450214608										
End Date	t	M	E	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α
01/31/2020	-1080	-0.2211199670	-0.2316101743	347.3179782	0.7691991069	169.6905537	4.799365263	189.1669724	12h 36m 40.07s	4° 47' 57.11"										
02/29/2020	-1051	-0.2151824872	-0.2253960293	347.65849	0.7686045448	169.3499935	4.933776752	189.4809991	12h 37m 55.43s	4° 51' 56.56"										
03/29/2020	-1020	-0.2090326251	-0.2187592967	348.0224075	0.7700229687	169.3899634	5.072767699	189.8167633	12h 39m 16.02s	5° 4' 38.27"										
04/30/2020	-990	-0.2029939381	-0.2123373381	348.3474542	0.7740031442	169.4383741	5.218693618	190.1464696	12h 40m 36.61s	5° 46' 43.8222"										
05/31/2020	-959	-0.1968346341	-0.2062872771	348.7307847	0.7780019191	169.4968297	5.368196184	190.4261914	12h 42m 0.00s	5° 49' 51.69"										
06/30/2020	-929	-0.2042130190	-0.2002721910	349.1691910	0.7819910017	169.5651937	5.523296184	190.7191977	12h 43m 18.33s	5° 52' 51.83"										
07/31/2020	-899	-0.1971166190	-0.1943100010	349.6100000	0.7859800000	169.6400000	5.683333333	191.0200000	12h 44m 36.66s	5° 55' 51.96"										
08/31/2020	-869	-0.1900000000	-0.1872000000	350.0500000	0.7899700000	169.7200000	5.843333333	191.3300000	12h 45m 55.00s	5° 58' 52.10"										
09/30/2020	-839	-0.1828833333	-0.1800833333	350.5000000	0.7939600000	169.8000000	6.003333333	191.6400000	12h 47m 13.33s	6° 01' 52.24"										
10/31/2020	-809	-0.1757666667	-0.1728666667	350.9500000	0.7979500000	169.8800000	6.163333333	191.9500000	12h 48m 31.66s	6° 04' 52.38"										
11/30/2020	-779	-0.1686500000	-0.1657500000	351.4000000	0.8019400000	169.9600000	6.323333333	192.2600000	12h 49m 50.00s	6° 07' 52.52"										
12/31/2020	-749	-0.1615333333	-0.1586333333	351.8500000	0.8059300000	170.0400000	6.483333333	192.5700000	12h 51m 8.33s	6° 10' 52.66"										
01/31/2021	-719	-0.1544166667	-0.1515166667	352.3000000	0.8099200000	170.1200000	6.643333333	192.8800000	12h 52m 26.66s	6° 13' 52.80"										
02/28/2021	-689	-0.1473000000	-0.1444000000	352.7500000	0.8139100000	170.2000000	6.803333333	193.1900000	12h 53m 45.00s	6° 16' 52.94"										
03/31/2021	-659	-0.1401833333	-0.1372833333	353.2000000	0.8179000000	170.2800000	6.963333333	193.5000000	12h 54m 3.33s	6° 19' 53.08"										
04/30/2021	-629	-0.1330666667	-0.1301666667	353.6500000	0.8218900000	170.3600000	7.123333333	193.8100000	12h 54m 51.66s	6° 22' 53.22"										
05/31/2021	-599	-0.1259500000	-0.1230500000	354.1000000	0.8258800000	170.4400000	7.283333333	194.1200000	12h 55m 10.00s	6° 25' 53.36"										
06/30/2021	-569	-0.1188333333	-0.1159333333	354.5500000	0.8298700000	170.5200000	7.443333333	194.4300000	12h 55m 28.33s	6° 28' 53.50"										
07/31/2021	-539	-0.1117166667	-0.1088166667	355.0000000	0.8338600000	170.6000000	7.603333333	194.7400000	12h 55m 46.66s	6° 31' 53.64"										
08/31/2021	-509	-0.1046000000	-0.1017000000	355.4500000	0.8378500000	170.6800000	7.763333333	195.0500000	12h 56m 5.00s	6° 34' 53.78"										
09/30/2021	-479	-0.0974833333	-0.0945833333	355.9000000	0.8418400000	170.7600000	7.923333333	195.3600000	12h 56m 23.33s	6° 37' 53.92"										
10/31/2021	-449	-0.0903666667	-0.0874666667	356.3500000	0.8458300000	170.8400000	8.083333333	195.6700000	12h 56m 41.66s	6° 40' 54.06"										
11/30/2021	-419	-0.0832500000	-0.0803500000	356.8000000	0.8498200000	170.9200000	8.243333333	195.9800000	12h 56m 60.00s	6° 43' 54.20"										
12/31/2021	-389	-0.0761333333	-0.0732333333	357.2500000	0.8538100000	171.0000000	8.403333333	196.2900000	12h 56m 58.33s	6° 46' 54.34"										
01/31/2022	-359	-0.0690166667	-0.0661166667	357.7000000	0.8578000000	171.0800000	8.563333333	196.6000000	12h 56m 56.66s	6° 49' 54.48"										
02/28/2022	-329	-0.0619000000	-0.0590000000	358.1500000	0.8617900000	171.1600000	8.723333333	196.9100000	12h 56m 55.00s	6° 52' 54.62"										
03/31/2022	-299	-0.0547833333	-0.0518833333	358.6000000	0.8657800000	171.2400000	8.883333333	197.2200000	12h 56m 53.33s	6° 55' 54.76"										
04/30/2022	-269	-0.0476666667	-0.0447666667	359.0500000	0.8697700000	171.3200000	9.043333333	197.5300000	12h 56m 51.66s	6° 58' 54.90"										
05/31/2022	-239	-0.0405500000	-0.0376500000	359.5000000	0.8737600000	171.4000000	9.203333333	197.8400000	12h 56m 50.00s	6° 61' 55.04"										
06/30/2022	-209	-0.0334333333	-0.0305333333	359.9500000	0.8777500000	171.4800000	9.363333333	198.1500000	12h 56m 48.33s	6° 64' 55.18"										
07/31/2022	-179	-0.0263166667	-0.0234166667	360.4000000	0.8817400000	171.5600000	9.523333333	198.4600000	12h 56m 46.66s	6° 67' 55.32"										
08/31/2022	-149	-0.0192000000	-0.0163000000	360.8500000	0.8857300000	171.6400000	9.683333333	198.7700000	12h 56m 45.00s	6° 70' 55.46"										
09/30/2022	-119	-0.0120833333	-0.0091833333	361.3000000	0.8897200000	171.7200000	9.843333333	199.0800000	12h 56m 43.33s	6° 73' 55.60"										
10/31/2022	-89	-0.0049666667	-0.0020666667	361.7500000	0.8937100000	171.8000000	10.003333333	199.3900000	12h 56m 41.66s	6° 76' 55.74"										
11/30/2022	-59	-0.0018500000	-0.0001500000	362.2000000	0.8977000000	171.8800000	10.163333333	199.7000000	12h 56m 40.00s	6° 79' 55.88"										
12/31/2022	-29	-0.0007333333	-0.0000333333	362.6500000	0.9016900000	171.9600000	10.323333333	200.0100000	12h 56m 38.33s	6° 82' 56.02"										
01/31/2023	1	0.0023833333	0.0016833333	363.1000000	0.9056800000	172.0400000	10.483333333	200.3200000	12h 56m 36.66s	6° 85' 56.16"										

UPRAN

TEP

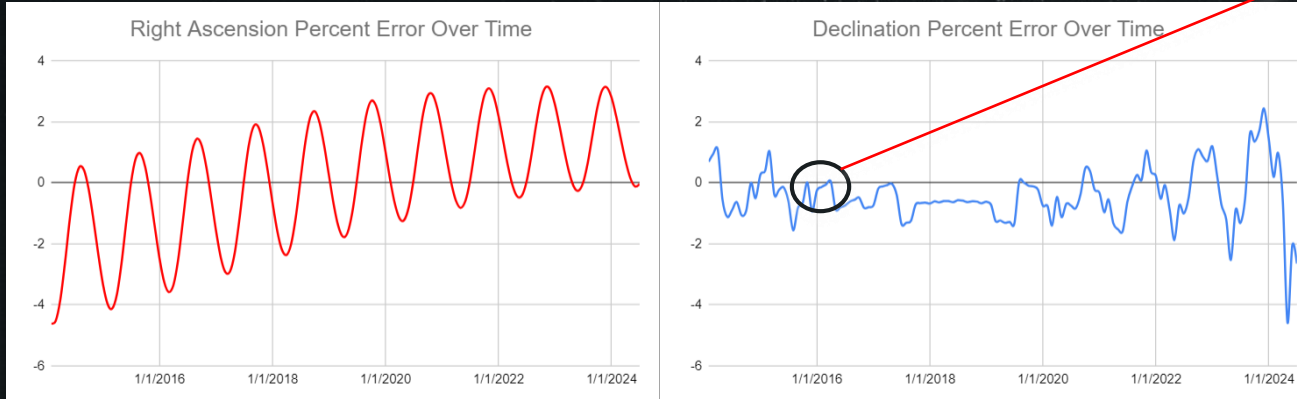
MAPS

WPN

578776

Application: Predicting Saturn's Orbit

Error Calculation



**MINOR
FLUCTUATION
AROUND 0%**

Explained by:

Gravitational
Forces from
Other Objects
(ex. Jupiter)

Effects of
General
Relativity

Linear Regression Models

- *Generated celestial coordinates for Saturn for each day 01/01/2014-05/31/2024*
- *Obtained actual values from an ephemeris*
- *Determined linear relationships between calculated data and actual observations (linearized error)*
- *Tested regression models on test data*
- *Compared models' predictions to ephemeris data*

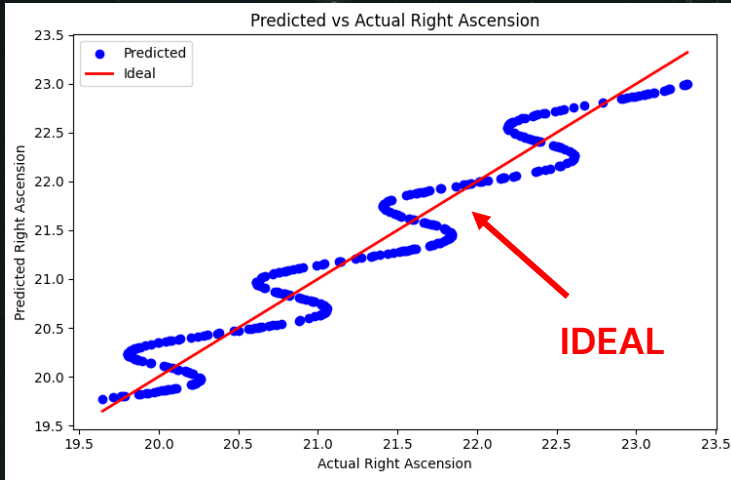
```
model_alpha = LinearRegression()  
model_alpha.fit(X_train, y_alpha_train)
```

```
model_delta = LinearRegression()  
model_delta.fit(X_train, y_delta_train)
```

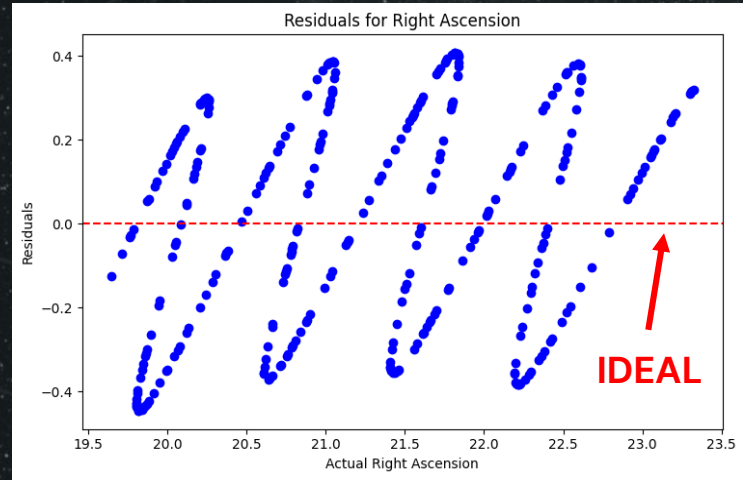
```
y_alpha_pred = model_alpha.predict(X_test)  
y_delta_pred = model_delta.predict(X_test)
```


Model Results

Right ascension (α)



Predictions vs Actual Ephemeris Data

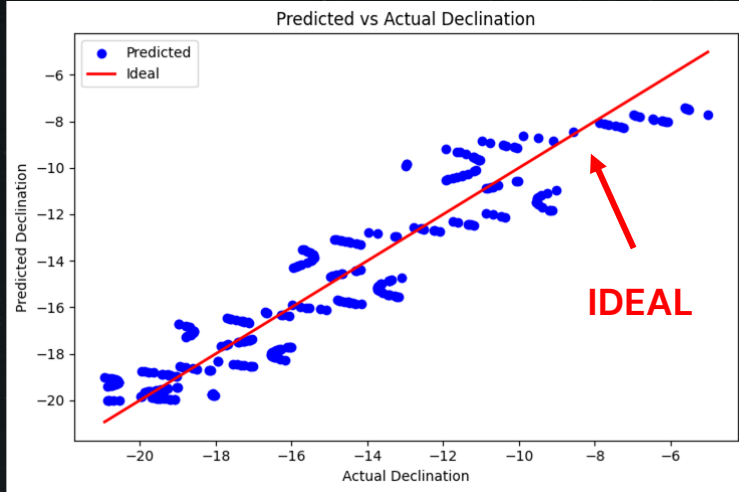


Residual Plot

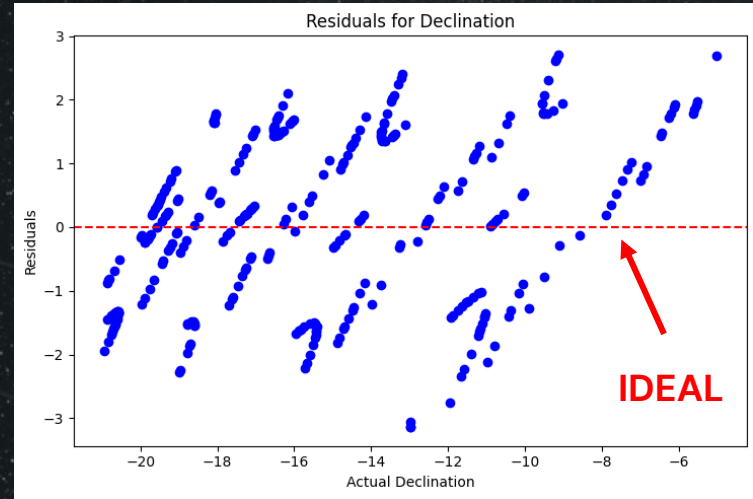
$R^2: 0.9276$

Model Results

Declination (δ)



Predictions vs Actual Ephemeris Data



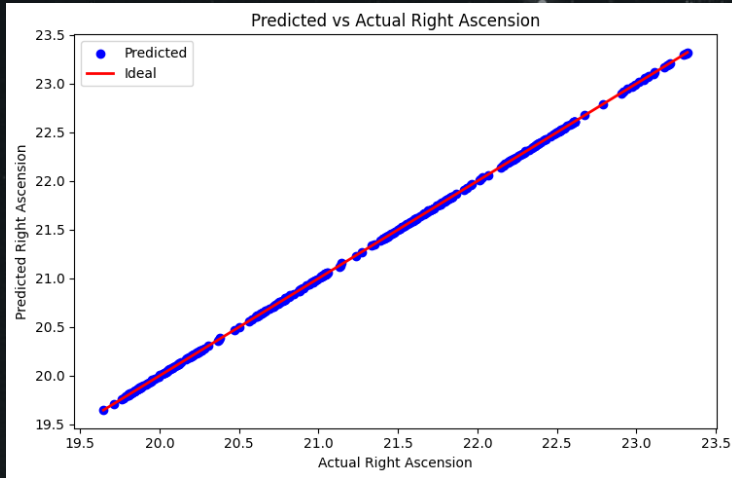
Residual Plot

$R^2: 0.8895$

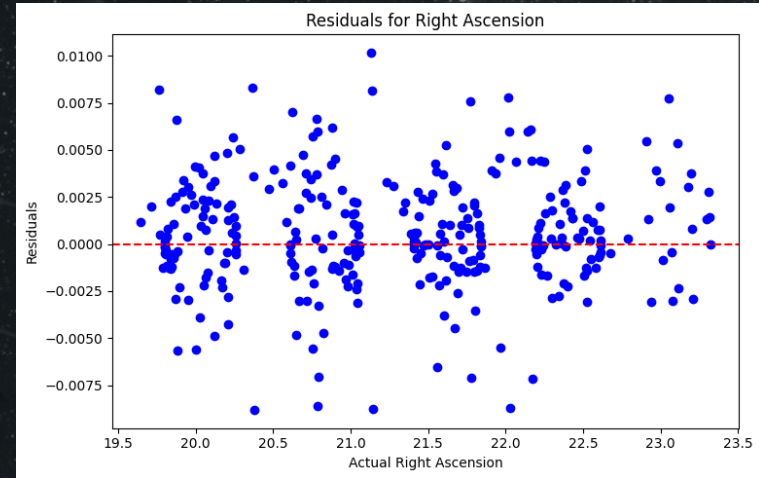
Random Forest Models

Using more complex models to account for underlying periodic trends

Right ascension (α)



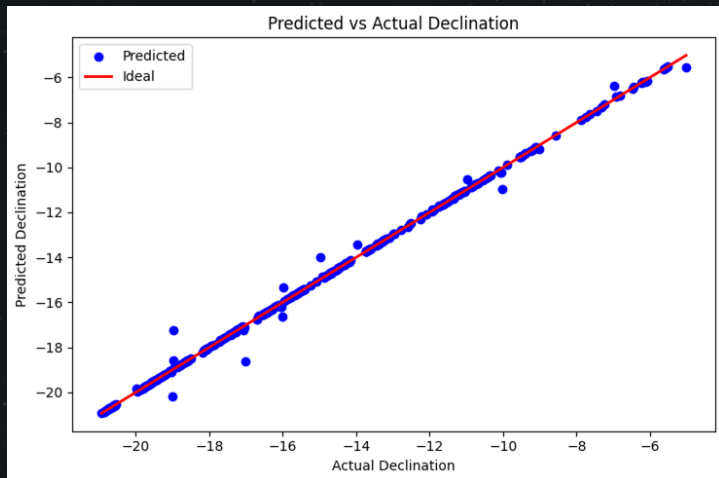
Predictions vs Actual Ephemeris Data



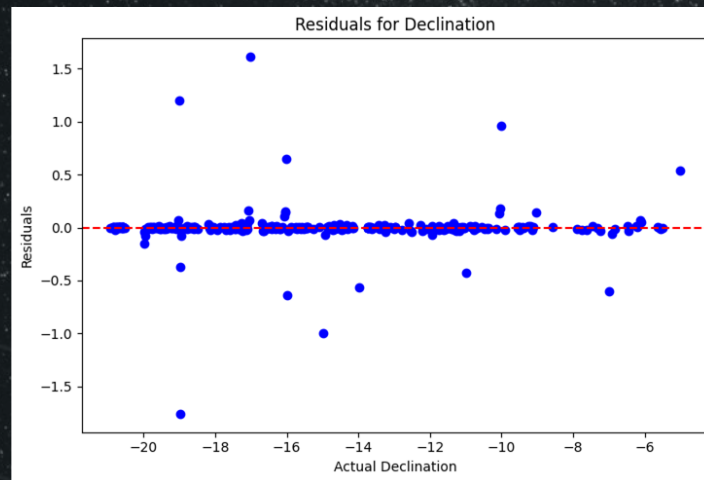
Residual Plot

$R^2: 0.99$

Declination (δ)



Predictions vs Actual Ephemeris Data



Residual Plot

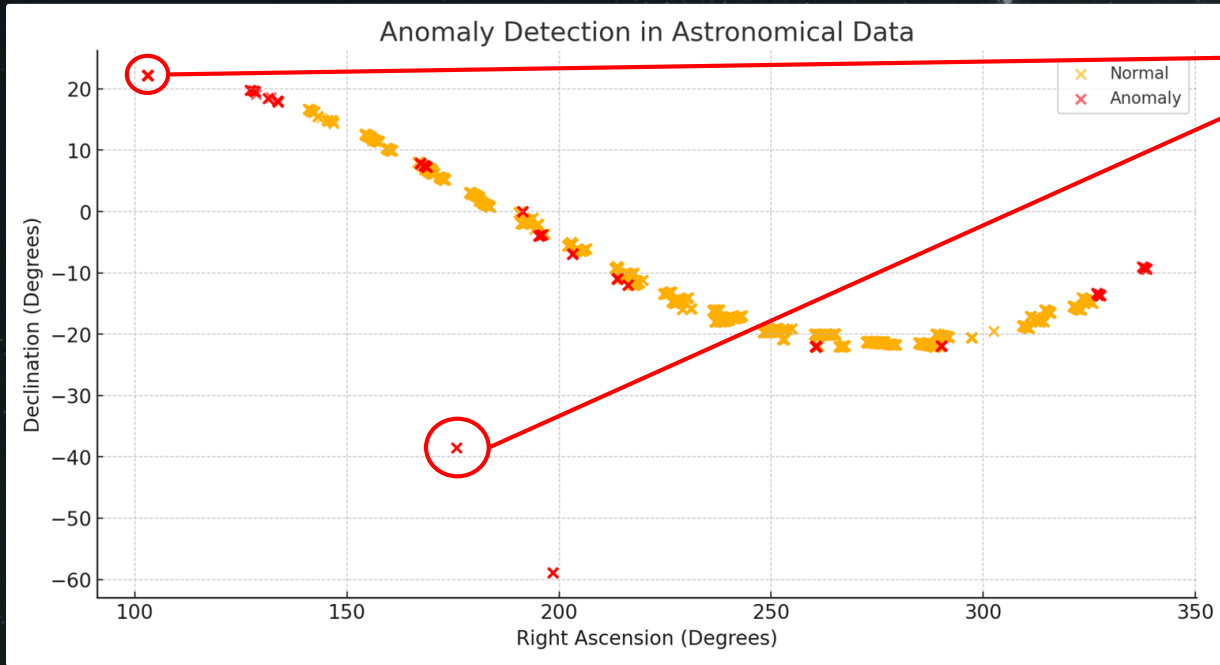
$R^2: 0.99$

09 ✨

Machine Learning - Anomaly Detection

The background is a dark, starry space scene. In the lower right, there is a faint image of a planet with rings, similar to Saturn. A constellation of stars is visible in the center, with lines connecting them to form a geometric shape. The overall aesthetic is futuristic and scientific.

One-Class Support Vector Machines



**ANOMALIES
DETECTED**

Radial Basis
Function Kernel

Explained by:

Unusual
celestial events

Observational
or recording
error

10



Conclusion



Machine Learning Integration

Linear regression and random forest models increased prediction precision.

One-Class SVM detected anomalies in planetary orbits, indicating unusual events or errors

Key Insights

Classical mechanics and modern techniques to enhance astronomical predictions.

Future work could involve training more complex models for improved accuracy.

Extend techniques to predict other future celestial phenomena and objects.

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Thank You!



Q & A



In loving spirit
of William
Colangelo III &
Tyler Harms

