Celestial Mechanics: Application of Kepler's Laws and Spherical Trigonometry

William Colangelo III, Adrien Cristian, Stefano D'Agostino, Mayank Deoras, Rishay Gupta, Tyler Harms, Jeffrey Jiang, David Ji, Aditya Kirubakaran, Krish Shah, Timothy Torubarov, Kevin Zhang

> Advisor: Steve Surace Assistant: Clifford Wijaya

Introduction

Table of Contents

Kepler's Laws

Newton's Universal Law of Gravitation

Kepler's Three Laws of Planetary Motion Trigonometry

02

Law of Sines and Cosines

Trigonometry of Spherical Geometry Polar Ellipses

03

Rectangular Ellipses U

Polar Representation of Shifted Ellipse O4 Connection Processes (01), (02), (03)

Right Ascension and Declination for Celestial Objects

Kepler's Laws

Kepler's First Law

Planets orbit in elliptical paths with their star as a focus.

Kepler's First Law

- Planets orbit around their host star
- These orbits are elliptical in nature
- The star occupies one focus of this ellipse



$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

r is the distance from the planet to the star, *a* is the length of the orbit's semi major axis and *e* is the eccentricity of the orbit.



Kepler's Second Law

The radius from the star to the planet sweeps out equal areas in equal amounts of time.

Kepler's Second Law

As a planet moves around its star, the radius between the bodies sweeps out an area which remains constant per unit time.

The change in area is given by:

$$\mathrm{d}A = \frac{1}{2}r^2 \left(\frac{h}{r^2}\right) \mathrm{d}t = \frac{h}{2} \mathrm{d}t$$



Where h is a constant such that $h^2 = GMa(1 - e^2)$ and r is the distance between the planet and its star.

Kepler's Third Law

The square of the period of a planet's orbit is proportional to the cube of its semi-major axis.

Kepler's Third Law

- The area of an ellipse is πab
- a is the length of the semi-major axis and b is the length of the semi-minor axis.
- Let T represent the period of the planet's orbit

The area can be calculated by integrating dA from 0 to T. From Kepler's 2nd Law, dA = (h/2)dt. After integrating and solving for T², the result is:

$$T^2 = \frac{4\pi^2 a^2 b^2}{h^2}$$

Since $h^2 = GMa(1 - e^2)$, the final result is: I

$$T^2 = \frac{4\pi^2}{GM}a^2$$

Conservation of Energy

 $E = \frac{1}{2}mv^2 - \frac{GMm}{r}$ $v^{2} = \left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^{2} + r^{2} \left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^{2}$ $r = \frac{a(1-e^2)}{1+e\cos\theta}$ $E = -\frac{GMm}{2a}$

Energy is conserved.

Ellipse Geometry

Ellipse Geometry





Connecting E to θ

 $\cos E = \frac{\cos \theta + e}{1 + e \cos \theta}$

 $r\sin\theta = b\sin E$





Showing that $\frac{dM}{dt}$ is constant

$M = E - e \sin E$

$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{h}{ab} = \frac{2\pi}{T}$



Now we've connected time to θ

Spherical Trigonometry

Spherical Trigonometry

- Geometry on a spherical surface has different rules
- Shortest path between two points lies on a "Great Circle"
- Spherical triangles are formed by these arcs

Spheric	al Law of	Sines
sinA	sin <i>B</i>	sinC
sina -	sinb	sinc

Spherical Law of Cosines $\cos c = \cos a \cos b + \sin a \sin b \cos C$



Celestial Sphere

A Shift in Reference Frame.....

Principles

- An apparent sphere formed by the sky (and its other half below the horizon)
- Rotates East to West
- Longitude, Latitude → Right Ascension, Declination

Assumptions

• Arbitrarily large distances



Points of Reference

Equatorial Coordinates

- Celestial Equator
 Vernal Equinox
 Celestial Poles
 Ecliptic Coordinates
 - Ecliptic
 - Vernal Equinox
 - Ecliptic Poles



Rise & Set Times

Sunrise and Sunset

declination determines the \bullet path of the sun in the sky one hemisphere visible from a given point on the Earth • time the sun spends in that hemisphere is the time the sun is in the sky highest point is noon subtract off for sunrise \bigcirc add on for sunset



Saturn Rise and Saturn Set

Given *θ* for Saturn, we can calculate "ecliptic" coordinates

 Given ecliptic coordinates, we can calculate Right Ascension and Declination

Given RA and Decl., we can calculate time spent in the sky.



Machine Learning – Error Analysis

Generating Data: Kepler's Laws

Saturn Constants										
<u> </u>	**			T (daya)		-	Last Perihelion			
113.665	339.392	2.485	0.0565	10755.7	23.5		July 26, 2003			
dM/dt										
0.0005841726068										
End Date	t	M	E	θ	β	λ	δ	α	α (formatted)	δ (formatted)
01/31/2020	603	3 524313337	3 504268424	201 9603639	-0.05863013754	295.0160926	-21 24030131	296 9808229	19h 47m 55 s	22° 45'34.92"
02/29/2020	6062	3.541254343	3.520361928	202.9321565	-0.1007428136	295.9869733	-21.10295526	298.0124981	19h 52m 3s	-22 53'49.36"
03/31/2020	6093	3.559363693	3.53757102	203.9709312	-0.1457258698	297.024776	-20.94925723	299.1131247	19h 56m 27.15s	-21° 3 2 67"
04/30/2020	6123	3.576888872	3.55423082	204.9761583	-0.1892108143	298.0290663	-20.79381502	300.1760076	20h 0m 42.24s	-21° 12'22.27"
05/31/2020	6154	3.594998222	3.571452234	206.0148515	-0.2340822446	299.066797	-20.62633884	301.2719292	20h 5m 5.26s	-21° 22'25.18
06/30/2020	6184	3.612523401	3.588124442	207.0199972	<u>-0 27743147</u> 09	300.0710176	-20.45770597	302.3301151	20h 9m 19.23s	-21° 32'32.26"
07/31/2020	6215	3.630632751	3.605 917.	208 6039	-0 22134 75	301 0867	276 224	303.4210427	20h 13m 41.05s	-21° 43'23.69"
08/31/2020	6246	3.648742102	3.6226	20 09 647	-0 \$6729 48	2	2 8 7063	304.5093323	20h 18m 2.24s	-21° 54'39.35"
09/30/2020	6276	3.66626728	3.639 379	38	-0 097692 57	J3. 04	19.5 741	305.5599462	20h 22m 14.39s	-20° 5'55.98"
10/31/2020	6307	3.684376631	3.656550+00	211.1406401	-0.454109755	304. 1079620	-19.70044719	306.6428696	20h 26m 34.29s	-20° 17'58.39"
11/30/2020	6337	3.701901809	3.67325803	212.1455676	-0.4968762776	305.1920082	-19.50008957	307.6881875	20h 30m 45.16s	-20° 29'59.68"
12/31/2020	6368	3.72001116	3.690530509	213.1839371	-0.5409058174	306.2294865	-19.2867762	308.765541	20h 35m 3.73s	-20° 42'47.61"
01/31/2021	6399	3.738120511	3.707811383	214.2222555	-0.5847558655	307.2669286	-19.06717495	309.8399969	20h 39m 21.6s	-20° 55'58.17"
02/28/2021	6427	3.754477344	3.723427336	215.1600469	-0.6241961135	308.2039424	-18.86349986	310.8079502	20h 43m 13.91s	-19° 8'11.4"
03/31/2021	6458	3.772586695	3.740724873	216.1982657	-0.6676648527	309.2413168	-18.63219527	311.8767875	20h 47m 30.43s	-19° 22'4.1"
04/30/2021	6488	3.790111873	3.757473099	217.2029426	-0.709521031	310.2451939	-18.4026313	312.9082918	20h 51m 37.99s	-19° 35'50.53"
05/31/2021	6519	3.808221224	3.7747888	218.2410554	-0.7525412631	311.2824994	-18.15960076	313.9712049	20h 55m 53.09s	-19° 50'25.44"
06/30/2021	6549	3.825746402	3.791555061	219.2456284	-0.7939369193	312.2863106	-17.9188773	314.9969317	20h 59m 59.26s	-18° 4'52.04"
07/31/2021	6580	3.843855753	3.808889868	220.2836327	-0.8364540757	313.3235487	-17.66451245	316.0538375	21h 4m 12.92s	-18° 20'7.76"
08/31/2021	6611	3.861965104	3.826234741	221.321581	-0.8786948745	314.3607534	-17.40454089	317.1076695	21h 8m 25.84s	-18° 35'43.65"
09/30/2021	6641	3.879490282	3.843029911	222.3259933	-0.9192969184	315.3644683	-17.14771599	318.1245724	21h 12m 29.9s	-18° 51'8.22"
10/31/2021	6672	3.897599633	3.860395286	223.3638302	-0.9609536546	316.4016085	-16.87701926	319.1723357	21h 16m 41.36s	-17° 7'22.73"
11/30/2021	6702	3.915124811	3.877210738	224.3681341	-1.000964864	317.4052622	-16.61001468	320.1833619	21h 20m 44.01s	-17° 23'23.95"
12/31/2021	6733	3.933234162	3.894597523	225.4058583	-1.041984809	318.4423402	-16.32900694	321.2250568	21h 24m 54.01s	-17° 40'15.58"
01/31/2022	6764	3 951343513	3 911995533	226 443525	-1.082661201	319 4793875	-16.0429201	322 263678	21h 29m 3 28s	-17° 57'25 49"

INPUTTED CONSTANTS

For each planet, constants were found and inputted

PROGRAMMED CALCULATIONS

Used mathematical equations derived previously to generate hundreds of data values from many times

Generating Data: Kepler's Laws

Leanue Constants					_							Jupiter Constan	nts											
O				T (days)	,	1	ast Peribelion					Ω	0	1	e	T (days)	c		Last Perihelion					
74.006	96 998	0.773	0.0457	30 689	215		anuary 15 2023					100.464	273.867	1.303	0.0489	4,332.59	3.13		January 23, 2023	1				
Midt												dM/dt											1000	
0.0002047407109											Last Peri	0.001450214	608											
											July													
End Date t		M	E é	9	β)	δ	a	a (formatted) & (formatted)			End Date	1	M	F	8	6	λ	δ	a	a (formatted)	δ (formattee	0	
01/31/2020	-1080	-0.2211199678	-0.2316101743	347 3179782	0.7691991069	169.6905357	4 799365263 189	1669724 12h 36m 40	07s 4* 47'57.71"			End Date			-		P				G (IOIIIIDAGO)	0 (1011101000		
02/29/2020	-1051	-0.2151824872	-0.2253960923	347 65849	0.7696405448	169.3499935	4.933767752 189.	4809691 12h 37m 55	43s 4' 56'1.66"			01/31/2	020 -1	088 -1.5778334	93 -1.6266572	18 269.60174	141 -0.0/88299/951	283.9318493	-3.116659605	283.952890	14 18h 55m 48.6	95 -4. 530.03		
03/31/2020	-1020	-0.2088355251	-0.2187529967	348.0224875	0.7700823667	168.9859634	5.077276999 189	8167633 12h 39m 16	02s 5* 4'38.2"			02/29/20	020 -1	059 -1.535///.	27 -1.5846725	62 272.00851	133 -0.1333733692	286.3380047	-3.13684644	286.36316.	57 19h 5m 27.16	\$ -4" 51 47.35		
04/30/2020	-990	-0 2026933038	-0.2123237581	348 3747452	0.7704803442	168.6336741	E 216002618 100	1410000 10h 40m 24	05+ 51 17:57 57			03/31/2	020 -1	028 -1.4908206	17 -1.5396969	71 274.58125	98 -0.1914157068	288.9101067	-3.152299085	288,93974	14 19h 15m 45.5	4s -4" 50'51.72		
05/31/2020	-959	-0.1963463418	0.205679774	348.7387469	0.7708509924	168.2696397	Mars Constants											291.3992704	-3.16119692	291.433283	31 19h 25m 43.9	9s -4" 50'19.69		
06/30/2020	-929	0204120	0.19	49.09100	77119973	167.91 3	Ω	0	1	e	T (days)	e		Last Perihelion				293.971469	-3.164114804	294.010010	07 19h 36m 2.4s	-4° 50'9.19"		
07/31/2020	-898	- 3857158	0.19 48813	19.4550	7151915	17.55 9	76.68	54.884	3.394	0.0067	224.701	23.5		October 29, 2016				296.4607818	-3.160858611	296.503679	07 19h 46m 0.88	s -4° 50'20.91		
08/31/2020	-867	-07510196	-0 1	49.819	180742	18 5												200.0224013	2 151217325	000 080527	78 19h 56m 19.3	3s -4° 50'55.62		
09/30/2020	-837	7136797	0.17 0420	350.17	05673	16 1 15	dM/dt											3 5057503	-3.1352152	5742	26 20h 6m 37.78	s -4° 51'53.23	(
10/31/2020	-506	NO.10	0.17 24428	350.5 998	0.7 5365	166 4	0.0279624269	9										3 0954913	-3 1137020	30 1238	37 20h 16m 36.3	s -4° 53'10.67	1000	
11/30/2020	-776	0.1535318306	-0.10.00503	351 35159 45	0.7725303737	166.12												3 5684224	-3.005286	128212	23 20h 26m 54.7	7s -4° 54'52.96	r	
01/31/2020	.743	-0.1325318250	-0.1530536207	351,2515646	0.7727736992	165 1025256	End Date	1 1	4 E		8	6	λ	δ	a	g (formatted)	δ (formatted)				1.2	s -4° 56'53.31		
02/28/2021	.686	-0.1404521277	-0 1471527653	351 9443905	0.7728683051	165.0637058	04/04/000	4400	22.24722500	22 25 27 20 24 7	101034447	4 470 400505	71220202000	40 30037000	224 40622002	451 27- 50.00	101 1020 001				.7	8s -4° 59'19.51	2. C.	
03/31/2021	-655	-0 1341051657	-0 1405051444	352 3084093	0 7729433589	164 6996538	01/31/202	1109	33.24132569	33.253/661/	404.9311117	1.170490505	230.52/664/	-10.28927008	234.4962263	10h 50m 40a	5 -12 42 30.03				0.2	75 -3" 27.6"		
04/30/2021	-625	-0.1279629443	-0.1340716807	352 6606869	0.7729862838	164.3473441	02/29/202	1218	34.05623607	34.00142523	511.3926429	-1.501836523	202.916/368	-24.36580252	204.200012	1011 0000 485	-25 30 3.11				95	-3° 4'57.97"		
05/31/2021	-594	-0.1216159823	-0.1274234901	353.0247085	0 7729999393	164.0287105	03/31/202	1249	34.92507131	34.92267875	201.05961/1	-3 292260369	332.5999009	-13.64279308	335.7949062	22h 23m 10.78	s -14° 21'25.94"				35	s -3° 8'26.77"		
06/30/2021	-564	-0.115473761	-0.1209895012	353 3769886	0.7729634436	164.3810227	04/30/202	12/9	35.76394412	35.75769902	249.1234944	-2.812990065	312.6258558	-19.75802308	315.9475062	21h 3m 47.4s	-20" 14'31.12"				92	s -3° 12'8.66"	Ø	
07/31/2021	-533	-0.1091267989	-0.1143407947	353.7410125	0.7729356975	164.7450798	05/31/202	0 1310	36.63077936	36.62488858	298.7885224	-0.3738399856	262 9964691	-23.68756058	262.3486068	17h 29m 23.67	s -24" 18'44.78"					95 -3" 16'17 98	e	
08/31/2021	-502	-0.1027798369	-0.1076918469	354 1050377	0.7728567489	165.109138	06/30/202	0 1340	37.46965217	37,46811824	346.8526443	2 258675908	214 9933163	-11 09024764	213.4715201	14h 13m 53.16	s -12° 54'35.11"					15 -3" 20'38 17	-	
09/30/2021	-472 -	-0.09663761556	-0.1012571645	354.4573211	0.7727506411	165.4614535	07/31/202	0 1371	38.3364874	38 9605	195422	9298053	0077	1478686	189.6105798	12h 38m 26.54	s 7° 50'32.92"		m (formatted)	X (formatted)		78 -3" 25'26 05	e	
10/31/2021	-441 -	-0.09029065352	0.09460778226	354.8213483	0.7726103042	165.8255137	08/31/202	0 1402	39.20332264	39 63	850042	2 198011	217.1 162	-12 12826212	216.1325741	14h 24m 31.82	s -13° 52'18.26"		a (lounatied)	o (ioimated)		a -3° 30'32 7"		
11/30/2021	-411 -	-0.08414843219	-0.0881727057	355.1736335	0.7724447972	166.177831	09/30/202	0 1432	40.04219545	40 59 8	49 48154	0.5. 52105	64065	7108608	265.3990126	17h 41m 35.76	s -24° 1'44.09"	323.4755163	21h 33m 54.12s	15° 52'11.3'		a -3º 35'46 7/	-	
12/31/2021	-380 -	-0.07780147015	0.08152294331	355.5376626	0 7722430929	166.541893	10/31/202	0 1463	40.90903069	4 185.	18 1488	1615	31 343021	-19 65223	318.8178567	21h 15m 16.29	s -20° 58'49.25"	323.3086728	21h 33m 14.08s	15° 56'8.48"	()	2- 2- 41'20 40		
01/31/2022	-349	-0.07145450811	0.07487300851	355.9016924	0.7720102131	166.9059558	11/30/202	0 1493	41.7479035	41 264713	23 793	-3.24, 550	329 752	/254142	333.0786152	22h 12m 18.87	s -15° 23'14.85"	323.1302019	21h 32m 31 25s	16° 0'21.51"	i i i i i i i i i i i i i i i i i i i	De 01 47146 01		
0.0279624269			_			_	12/31/202	0 1524	42.61473873	42.60816789	281.6444311	-1.351144492	280.1148441	-24.4605423	281.1211995	18h 44m 29.09	s -25° 32'22.05"							
							01/31/202	1 1555	43.48157397	43.47833857	331.3101348	1.497453578	230.5256831	-16.48283234	228.4899772	15h 13m 57.59	s -17° 31'1.8"		T (days)	e	L	st Perihelion		
End Date	t	M	E	0	β	λ	02/28/202	1 1583	44.26452193	44.26639991	376.1706323	3.209944061	185.6562761	0.693948615	186.4640683	12h 25m 51.38	s 0° 41'38.22"	0.2056	87.969	23.5		April 26, 2021		
01/31/202	21	61 489376	95 61 482743	36 283 0815	033 -1 272669	328 278 6775	03/31/202	1 1614	45.13135716	45.1374868	425.8367607	2.917264806	197.4449518	-4.169831678	197.1962216	13h 8m 47.09s	-5° 49'48.61"							
02/29/202	22	28 62.300287	33 62.296820	329 5429	495 1.40284	1957 232 2888	95 04/30/202	1 1644	45.97022997	45.97633877	473.8996756	0.6598085069	245.4828511	-20.62268113	243.6823262	16h 14m 43.76	s -21° 22'38.35"							
03/31/202	22	63.167122	63 169374	39 379 2099	385 3.263908	182.6105	91 05/31/202	1 1675	46.83706521	46.83894859	523.565812	-2.109729863	295.0808391	-23.24518044	297.4538448	19h 49m 48 92	s -24° 45'17.35"							
04/30/202	22	64.005995	38 64.012283	427.2738	615 2.872832	198.8812	01 06/30/202	1 1705	47.67593802	47.67244427	211 6306087	-3.387714892	343.1884998	-9.748137372	345.833358	23h 3m 20.01s	-10° 15'6.71"			-				
05/31/202	23	64.872830	64.8788	476.938	856 0.4824659	279 248.5150	04 07/31/202	1 1736	48 54277326	48 53615727	261 2962 368	-2 349262874	300 4495161	-22 39899743	303 2075531	20h 12m 49.81	s -23" 36'3.61"	E	8	p /	0		a (form	atted) 0 (formatte
06/30/202	23	65.711703	65 713451		-7-5736	701 96.000	54 08/31/202	1 1767	49 40960849	49 40452677	310 9614023	0 3454606477	250 8447795	-21 78565821	249 3070395	16h 37m 13 69	8 -22" 12"51 63"	236 -32.381786	12 313.8879675	-2.041711642	245.1994859	-23.22957334	242.8594258 16h 11	m 26.26s -24* 46'13
07/31/202	23	65.578538	66.5746	6698	936 - <mark>:</mark> 97	004 46. 1	09/30/202	1 1797	50 2484813	50 24836663	359.0258837	2 742105545	202 8180079	6 349173508	202 1244704	13h 28m 29 87	-7º 39'2 98"	152 -29.936607	433.053/11/	6.845633102	150.5975071	17.6785078	204.7894575 13h 35	m9.4/s 1/*4042
08/31/202	24	67.4453	39 57.438	8 3354	141 - <mark>b</mark>	105 97.4	45 10/31/202	1 1828	51 11531654	51 12037168	408 6925389	3 299055687	180 2795084	2 913747975	181 5728995	12h 6m 17 5s	2" 54'49 49"	27 -27 305030	2 200.3000444	-5.314273053	277.5434959	-20.50901710	270.5612105 108 34	m 14.095 -29" 24.30
09/30/202	24	42 68 284246	71 279	08	0. 986	2 49 4	47 11/30/202	1858	51 95418935	51 96083744	456 7556152	1 511473642	228 36164	16 78072130	226 3535849	15h 5m 24 86c	16' 17'38 8"	106 .23.36848	449 2040006	6 161766993	166.838628	10.87703986	189 6609768 12h 3	im 38.63s 10° 52'37
10/31/202	24	173 69.15108	96 💡 8	\$67 0 651	921 1 4312	8 <mark>2 99.7 1</mark>	12/21/202	1 1000	E2 021024E0	£2 824706E0	EDE 4211030	1 232634763	277 0611670	24 46100274	279 7411105	10h 34m 67.97	· 351 30'32 30"	2 5	15 101		290 8332	091	871 19h 3	9 115 -29* 54'34
11/30/202	25	69.989954	176 6. <u>9</u> 6	347	8013	875 81.71	01/21/202	1003	52.02102450	52.02470355	106 0891315	-1.232534763	277.5511575	16 22004207	220.0679130	701 3411 97.07	152 20/40 17	-19 3034	7 338.764 3	89518281	220 3854	098 09	2 18033 14h	31 285 -15' 54'6.
12/31/202	25	34 70.856785	199 70.863482	98 459 7947	829 1.450895	042 231.3956	44			energy of the			and the second se			VOIDTIEVEN		18/ 10/10/12	7 465.47 /2	985442451	183 5786	401 77	823291	m 17.59s 3° 20'24.6
01/31/202	25	1.72362	23 /1./2/060	34 509.4604	358 -1.398399	1081 280.9846	819 -24 43/905/2 100 - 44 04 7905/2	282 0775333 18h	05/31	/2021	-433 -0.0452	20051899 -0.045	59201877 3	57.4101056 -1.	769940	09/30/2020	- V (41 9	37 <mark>-14.991431</mark> (55 EE 9901	866310005	306. 8809	229 00	11.2168344 4	m 52.04s -26* 45'14
02/28/202	25	93 72.50657.	19 72.504902	194 3209	482 -3.172668	1252 325.8495-	429 -15.91772694	329.2318379 21h	06/30	2021	-403 -0.0420	6884331 -0.042	43323543	357.589544 -1.	769886	10/31/2020	-112.6 2	-12.661817	9 355.5 655	10908857	203. 938	5498 06	3.1167653 37	m 28.02s -7° 27'0.5"
03/31/202	26	24 73.373408	42 /3.36/31	155 243.9871	619 -2.9/1/46	14/ 31/./642	888 -18.37380474	321.1/90618 21h	07/31	2021	-372 -0.0388	3277844 -0.039	16915546 3	57.7749637 -1.	769812	11/30/2020	-147 -10.4 74	11 61	89 477.71 25	3.0.	195.36	-2.5343 93	6199375 2r	a 28.78s -3° 27'56.1
04/30/202	20	104 74.21220	23 74.205902	292 0500	513 -0.7668640	209.7218	150 -24.2005007	269.6946611 1/h	08/31/	2021	-341 -0.0355	9671357 -0.035	90507187 3	57.9603834 -	1.76971	12/31/2020	-116 -8.2852993	-8.454883	59 245.9765426	-6.976127679	313.1923137	-23 57455776	317.8376974 21h 11	m 21.05s -24' 25'31
05/31/202	20	100 75.079110	147 75.070303	71 300 7000	109 2.022029	1300 220.1200	239 -12.97005339	100.0003400 140	09/30	2021	-311 -0.032/	6503789 -0 032	74627815 3	58 1398218 -1	769612	01/31/2021	-85 -6.0711	245 -6.01705155	55 372.4026377	4.636933088	187.0172711	1.465860985	188.2765815 12h 33	m 6.38s 1° 27'57.1
07/31/202	27	46 76 78 493	52 76 791513	67 439 4460	025 2.427404	944 211 0220	813 9 5961022015	209 7712312 126	10/24	2021	-280 -0 0201	2897302 0 020	48218833 2	58 3252415	1 7604	02/28/2021	-57 -4.071224	-3.9259890	485.9973494	2.940525006	203.6157468	-0.458546919	202.9433557 13h 31	m 46.416 -7* 3229.2
08/31/202	27	77 77 651650	75 77 656913	489 111	722 -0 2363630	584 260 6667	363 .23.40638252	259 8213874 175	10/31	2021	-200 -0.0292	2097 302 -0.025	40210033 3	00.3232413	700040	03/31/2021	-20 -1.05/049	147 -2.04039370	254.2013169	-0.014525419	102 7266174	-25.6900647	104 40004 12h 1	m 225 -20° 10 35
09/30/202	28	107 78 49053	56 78 490.86	53 537 1761	824 -2 676109	617 308,6893	-20.71502079	311.8801261 205	11/30	2021	-250 -0.0260	19129134 -0.026	32338914 3	00.0046799 -1.	109340	05/31/2021	35 2 456874	794 2 6049832	502 5779213	1.008374813	220 0939917	-13 92390/16	217 9986107 146 3	im 59 67s -14" 4"331
10/31/202	28	38 79.35736	78 79 352430	41 226 8429	138 -3 323078	162 334 9310	536 12 81640774	338.0300806 22h	12/31	/2021	-219 -0.0228	86123247 -0.023	05929425 3	58.6900996 -1	769175	06/30/2021	65 4.642624	517 4 444 36532	266 4088347	-6 317040114	292 6308562	.27 82133538	295 6227265 196 4	/m 29 45s -28° 10'43
11/30/202	20	168 80.19674	61 80.189451	74 274 9060	187 -1.707014	633 286,8442	633 -24 12882807	288.5040633 196	01/31	/2022	-188 -0.019	6251676 -0.019	79519724 3	58.8755192 -1	768990	07/31/2021	96 6.856799	435 6.9903779	34 393.3587001	6.208329723	166.0245268	11.23947543	190.3998707 12h 4	m 35.97s 11* 14:22
12/31/202	28	89 81.063075	85 81.05911	21 324.5714	959 1 129867	366 237 2540	703 -18 49551802	235 2313216 15h	10m 55 52s -19* 30	016.14	I COLUMN					08/31/2021	127 9.070974	252 9.13059117	9 519.3118707	-1.025000761	236.7048151	-20.46575429	234.1374819 15h 34	im 33s -21* 32'3.7
01/31/202	29	30 81.92991	08 81.931594	165 374.2384	365 3.170928	187.589	077 -0.1062297925	188.2206317 12h	32m 52.95s -1" 53	37.57						09/30/2021	157 11 21372	11.0081403	282.5683145	-5.224298847	276.4254417	-28.56145125	277 2897302 18h 25	m 9.54s -29° 26'18
																10/31/2021	188 13.42789	13.6050740	409.7895768	6.872656547	149.4986885	18.09974923	205.847317 13h 43	m 23.36s 18* 5'59.1
																11/30/2021	218 15.57064	15 594024	33 531 9622601	-2.514386416	249.2735966	-24 38179384	247.1583553 16h 20	im 38.01s -25° 37'5.4

Application: Predicting Saturn's Orbit

Error Calculation



MINOR FLUCTUATION AROUND 0%

Explained by:

Gravitational Forces from Other Objects (ex. Jupiter)

Effects of General Relativity

Linear Regression Models

- Generated celestial coordinates for Saturn for each day 01/01/2014-05/31/2024
- Obtained actual values from an ephemeris
- Determined linear relationships between calculated data and actual observations (linearized error)
- Tested regression models on test data
- Compared models' predictions to ephemeris data

model_alpha = LinearRegression()
model_alpha.fit(X_train, y_alpha_train)

model_delta = LinearRegression()
model_delta.fit(X_train, y_delta_train)

y_alpha_pred = model_alpha.predict(X_test)
y_delta_pred = model_delta.predict(X_test)

Model Results

Right ascension (α)



Predictions vs Actual Ephemeris Data

Residual Plot

R²: 0.9276

Model Results

Declination (δ)



Predictions vs Actual Ephemeris Data

Residual Plot

R²: 0.8895

Random Forest Models

Using more complex models to account for underlying periodic trends

Right ascension (α)



Predictions vs Actual Ephemeris Data

Residual Plot

R²: 0.99

Declination (δ)





Predictions vs Actual Ephemeris Data

Residual Plot

Machine Learning – Anomaly Detection

One-Class Support Vector Machines



ANOMALIES DETECTED

Radial Basis Function Kernel

Explained by:

Unusual celestial events

Observational or recording error

Conclusion

Machine Learning Integration

Key Insights

Linear regression and random forest models increased prediction precision.

One-Class SVM detected anomalies in planetary orbits, indicating unusual events or errors Classical mechanics and modern techniques to enhance astronomical predictions.

Future work could involve training more complex models for improved accuracy.

Extend techniques to predict other future celestial phenomena and objects.

References

- 1. Newton, I. *Philosophiæ Naturalis Principia Mathematica*. London: Royal Society, 1687.
- 2. Smart, W. M. *Textbook on Spherical Astronomy*. Cambridge: Cambridge University Press, 1977.
- 3. Meeus, J. Astronomical Formulae for Calculators. 4th ed. Leuven: Willmann-Bell Press, 1988.
- 4. In-The-Sky.org. Ephemeris. *In-The-Sky.org*. https://in-thesky.org/ephemeris.php. Accessed 2024 Jul 17-26.
- 5. European Southern Observatory. 2023. ESO Science Archive Facility. https://archive.eso.org/eso/eso archive main.html. Accessed 2024 Jul 17–26.

Thank You!



In loving spirit of William Colangelo III & Tyler Harms

