AI-ASSISTED DISCOVERY OF PHYSICS LAWS

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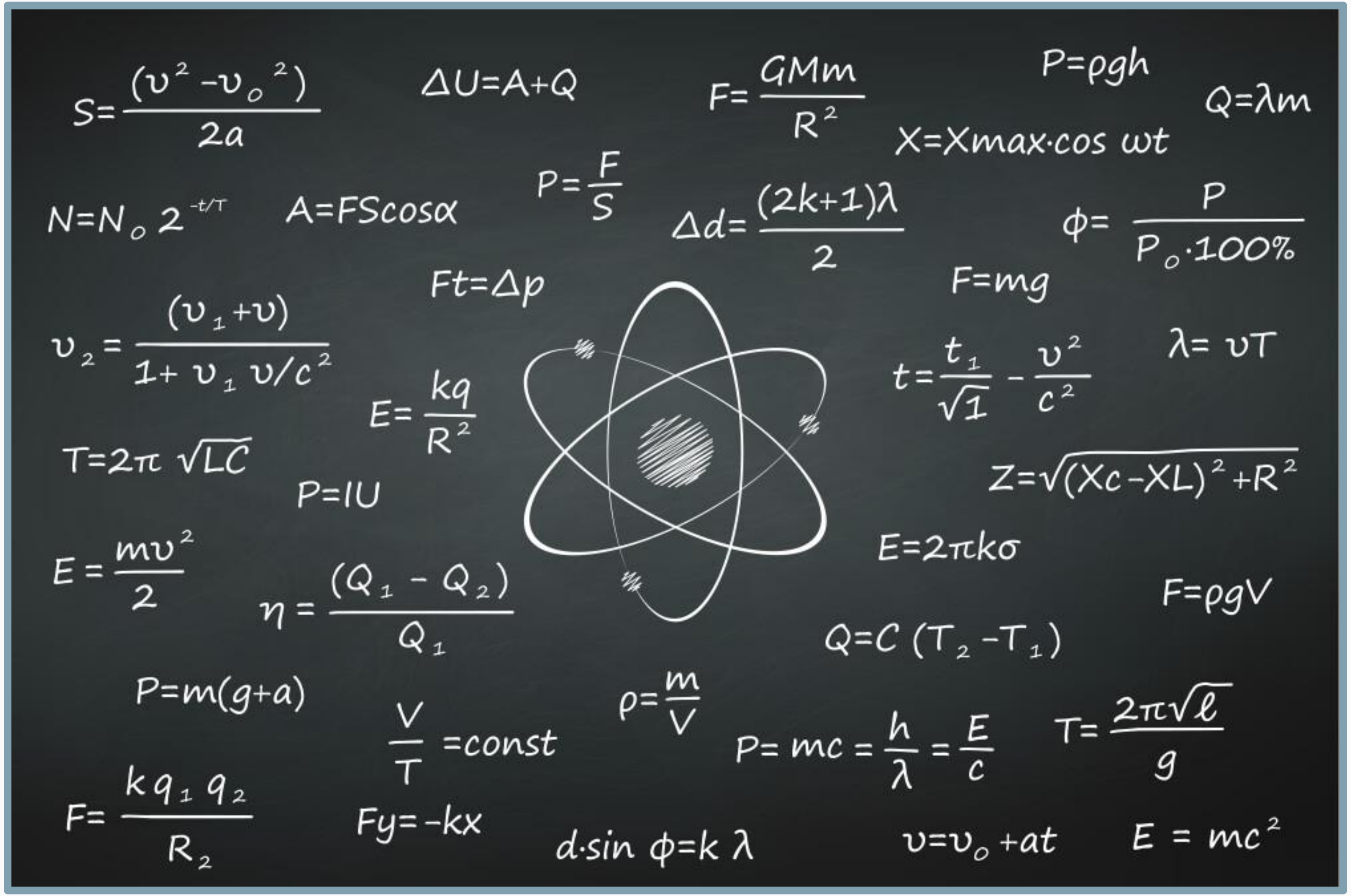
$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$


$S=\frac{(\upsilon^2-\upsilon_0{}^2)}{2a}$
$\Delta U=A+Q$
$F=\frac{GMm}{R^2}$
$P=\rho gh$
$Q=\lambda m$
$X=Xmax\cdot\cos\omega t$
$N=N_0 2^{-t/T}$
$A=FS\cos\alpha$
$P=\frac{F}{S}$
$\Delta d=\frac{(2k+1)\lambda}{2}$
$\phi=\frac{P}{P_0\cdot 100\%}$
$Ft=\Delta p$
$F=mg$
$\upsilon_2=\frac{(\upsilon_1+\upsilon)}{1+\upsilon_1\upsilon/c^2}$
$E=\frac{kq}{R^2}$
$t=\frac{t_1}{\sqrt{1}}-\frac{\upsilon^2}{c^2}$
$\lambda=\upsilon T$
$T=2\pi\sqrt{LC}$
$P=IU$
$Z=\sqrt{(Xc-XL)^2+R^2}$
$E=2\pi k\sigma$
$E=\frac{m\upsilon^2}{2}$
$\eta=\frac{(Q_1-Q_2)}{Q_1}$
$F=\rho gV$
$Q=C(T_2-T_1)$
$P=m(g+a)$
$\frac{V}{T}=const$
$\rho=\frac{m}{V}$
$P=mc=\frac{h}{\lambda}=\frac{E}{c}$
$T=\frac{2\pi\sqrt{\ell}}{g}$
$F=\frac{kq_1q_2}{R_2}$
$Fy=-kx$
$d\cdot\sin\phi=k\lambda$
$\upsilon=\upsilon_0+at$
$E=mc^2$

$y = ?$

x_0	x_1	x_2	x_3	y
3.1427	2.7183	1.4142	0.5772	7.5641
-4.0039	0.1234	2.3026	3.9999	6.8839
8.1812	-1.6180	3.3333	-2.7182	-12.3129
0.0001	5.5555	0.7071	1.0001	4.1784
1.6180	-3.1415	3.1415	2.3026	3.1112
2.2222	1.0101	4.4444	-1.1111	-4.2621

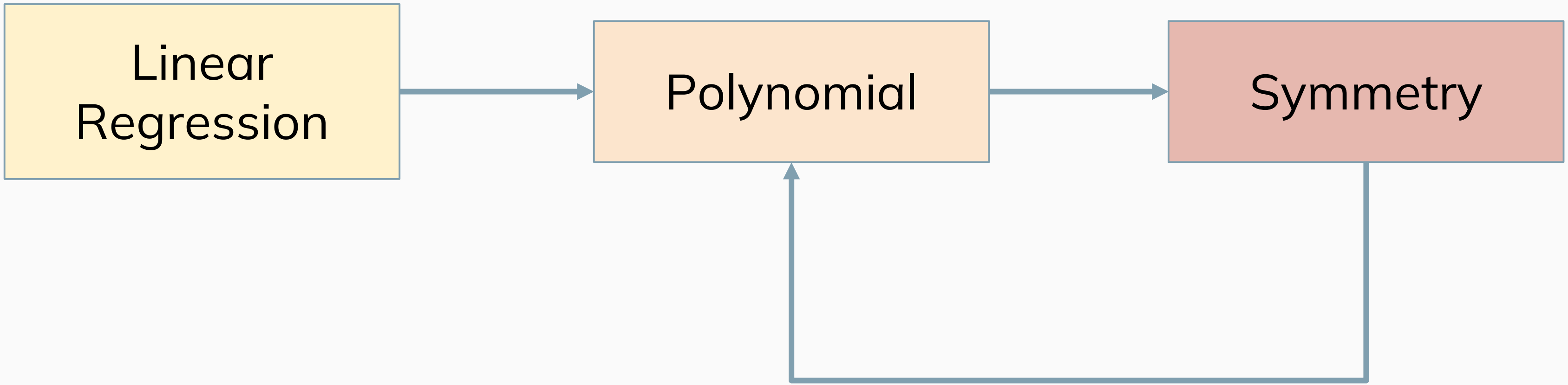
$$x_f = x_0 + v_0 t + \frac{1}{2} a t^2$$

x_0	v_0	t	a	x_f
3.1427	2.7183	1.4142	0.5772	7.5641
-4.0039	0.1234	2.3026	3.9999	6.8839
8.1812	-1.6180	3.3333	-2.7182	-12.3129
0.0001	5.5555	0.7071	1.0001	4.1784
1.6180	-3.1415	3.1415	2.3026	3.1112
2.2222	1.0101	4.4444	-1.1111	-4.2621

Let's Solve a New Mystery

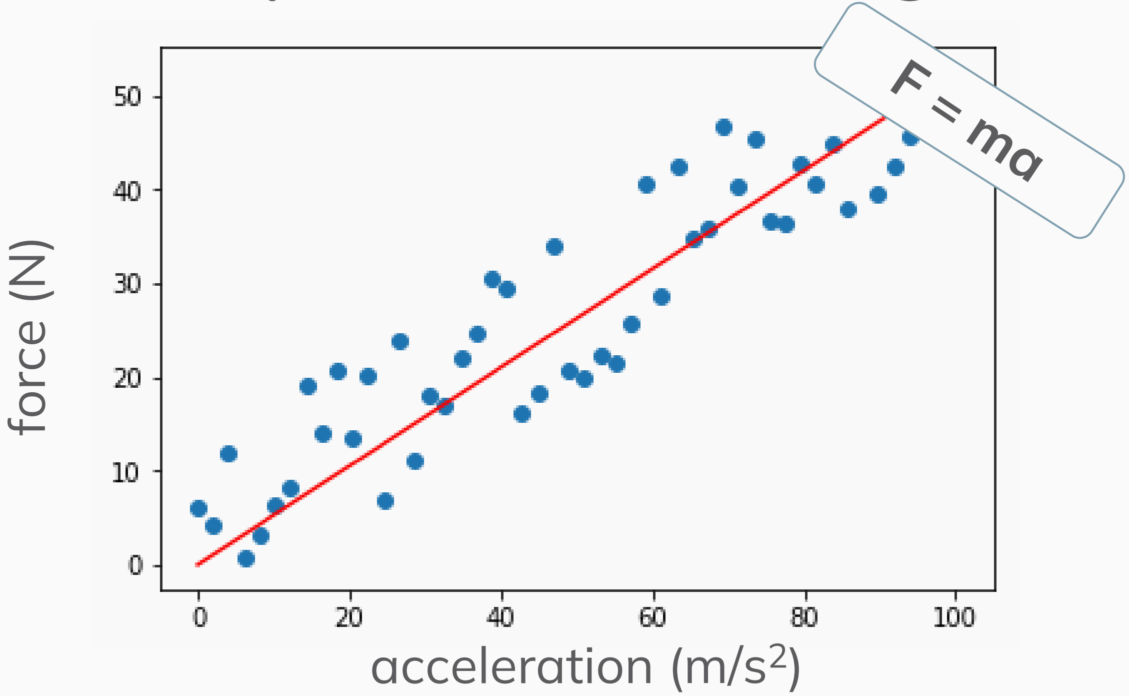
x_0	x_1	x_2	x_3	y
14	23	28	14	-448
17	30	11	16	589
15	19	11	14	739
16	14	17	14	172
20	23	16	20	584
11	11	21	18	55
28	18	30	18	352

Intro: Flow of Our Algorithm



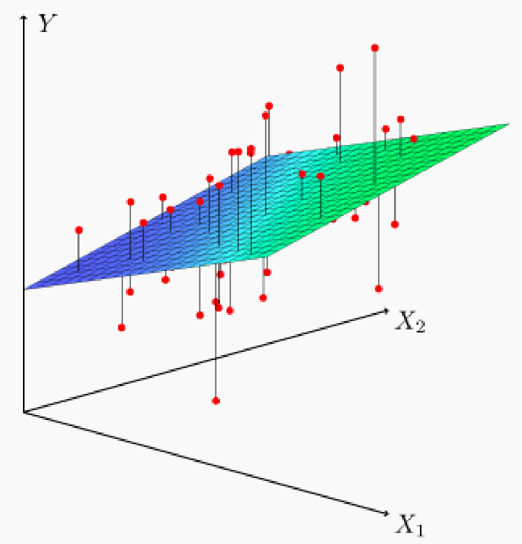
Inspiration from "AI Feynman" by Udrescu and Tegmark (2020)

First Step: Linear Regression



But how do we do this with multiple independent variables?

First Step: Linear Regression



$$y = \beta_0 + \beta_1 X_1 + ... + \beta_n X_n$$

Let's Try It On This!

x_0	x_1	x_2	x_3	y
14	23	28	14	-448
17	30	11	16	589
15	19	11	14	739
16	14	17	14	172
20	23	16	20	584
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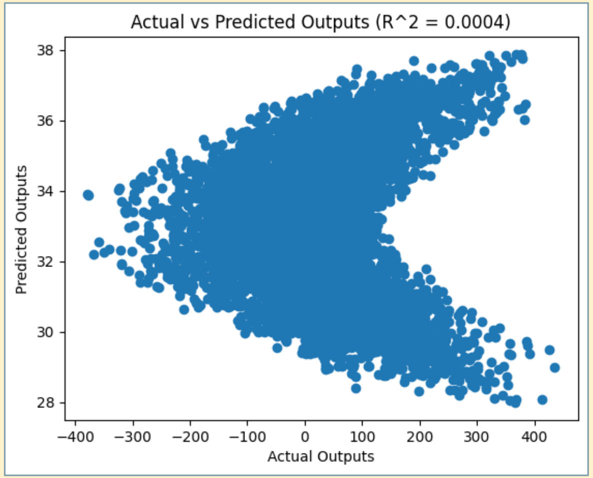
Testing Linear on the Mystery

Method	Output
LINEAR REGRESSION	

Predicted Value

Actual Value

Testing Linear on the Mystery

Method	Output
`LINEAR REGRESSION`	✗

Why could this have failed?
Maybe it's a <u>polynomial</u>!

Second Step: Polynomial Regression

Ex: $y_f = y_0 - 4.9t^2$

y_0	t	y_f
1	2	-18.6
3	4	-75.4
5	6	-171.4
7	8	-306.6

→

y_0	y_0^2	t	t^2	y_f
1	1	2	4	-18.6
3	9	4	16	-75.4
5	25	6	36	-171.4
7	49	8	64	-306.6

→

Linear Regression

Let's Try It On This!

x_0	x_1	x_2	x_3	y
14	23	28	14	-448
17	30	11	16	589
15	19	11	14	739
16	14	17	14	172
20	23	16	20	584
11	11	21	18	55
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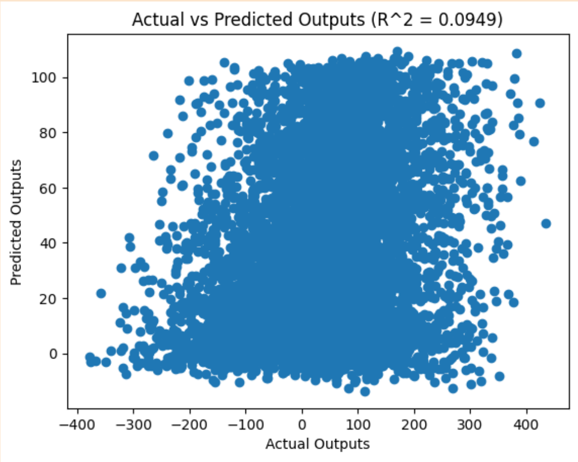
Testing Polynomial on the Mystery

Method	Output
LINEAR REGRESSION	✗
POLYNOMIAL REGRESSION	

Predicted value

Actual Value

Testing Polynomial on the Mystery

Method	Output
LINEAR REGRESSION	✗
POLYNOMIAL REGRESSION	✗

Why could this have failed?

Ex: $y = (x_0 - x_1)(x_2 - x_3) \longrightarrow x_0x_2 - x_1x_2 - x_0x_3 + x_1x_3$

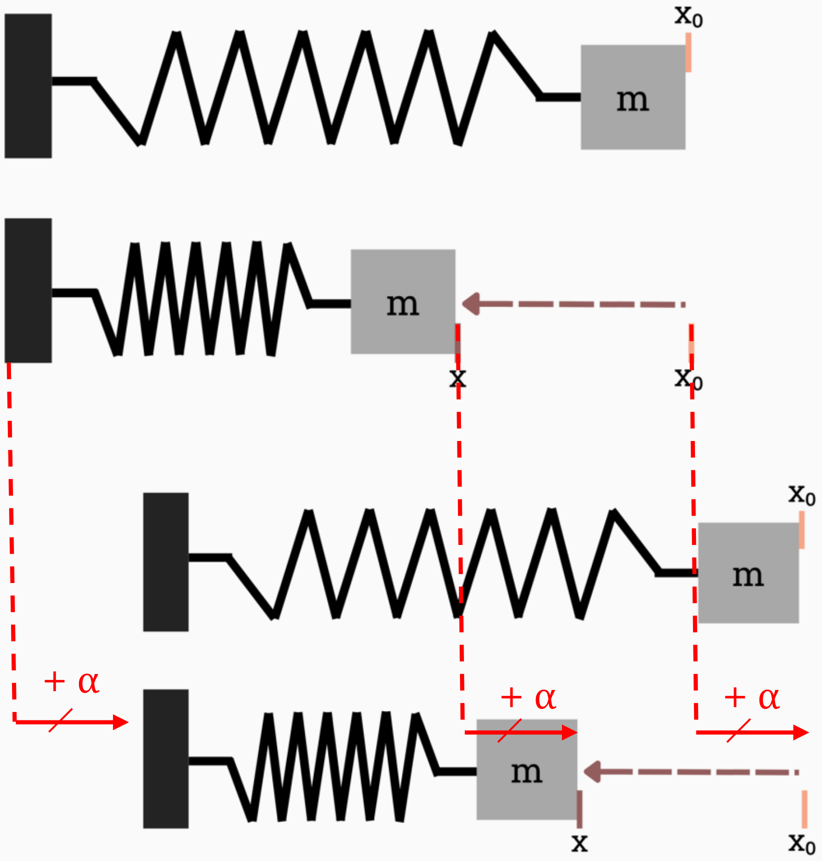
We could try…

x_0	x_1	x_2	x_3

x_0	x_1	x_2	x_3	x_0x_1	x_0x_2	x_0x_3	x_1x_2	x_1x_3	…

<u>Issue:</u> Too many combinations

Ex: $x_0x_1^2$, $x_0x_1^3$, $x_0x_1x_2^2$



Third Step: Symmetries

$$U_e = \frac{1}{2}k(x_0 - x)^2$$

Translational symmetry

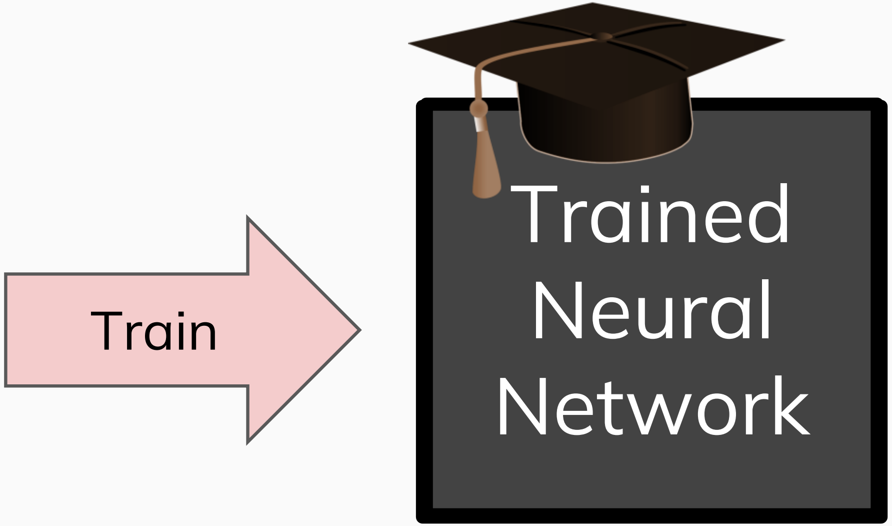
$$U_e = \frac{1}{2}k(x_0 - x)^2$$

$$U_e = \frac{1}{2}k[(x_0 + \cancel{a}) - (x + \cancel{a})]^2 \quad \Rightarrow \quad U_e = \frac{1}{2}k(x_0 - x)^2$$

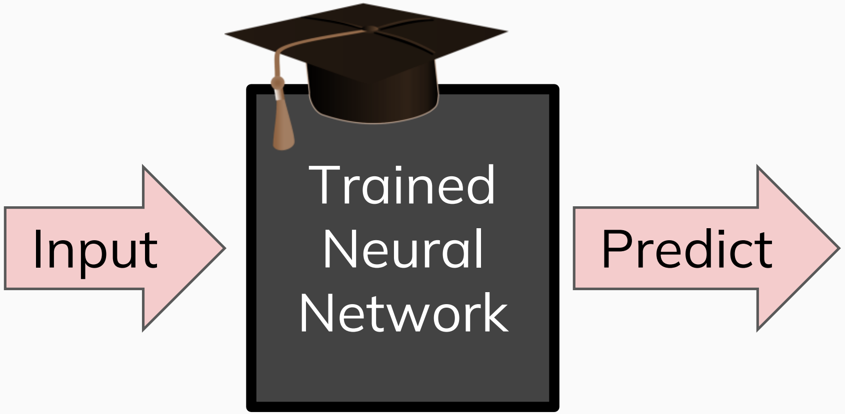
Individual values do not matter, but their <u>difference</u> does

Finding Symmetries Using Neural Network

x_0	x	k	U_e
5	2	0.5	4.5
6	3	2	18
4	2	1	4
0	-4	3	48



$x_0 + \alpha$	$x + \alpha$	k
$5 + \alpha$	$2 + \alpha$	0.5
$6 + \alpha$	$3 + \alpha$	2
$4 + \alpha$	$2 + \alpha$	1
$0 + \alpha$	$-4 + \alpha$	3



U_e (predicted)
4.7
18.4
3.5
45.2

U_e (predicted)
4.7
18.4
3.5
45.2

Compare

Close enough!

U_e (original)
4.5
18
4
48

x_0	x	k	U_e
5	2	0.5	4.5
6	3	2	18
4	2	1	4
0	-4	3	48

Reduce

$(x_0 - x)$	k	U_e
3	0.5	4.5
3	2	18
2	1	4
4	3	48

Let's Try It On This!

x_0	x_1	x_2	x_3	y
14	23	28	14	-448
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15	19	11	14	739
16	14	17	14	172
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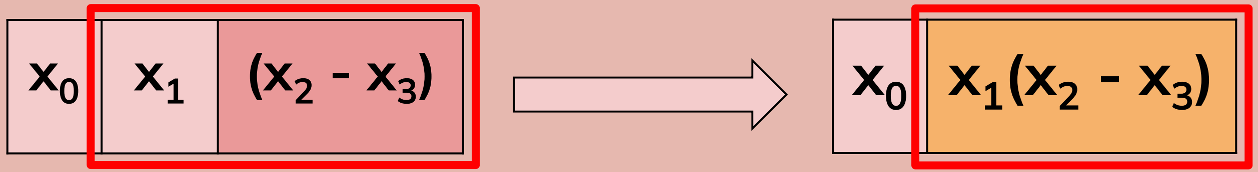
Testing Symmetries on the Mystery

Method	Output
LINEAR REG.	✗
POLYNOMIAL REG. #1	✗
SYMMETRY SEARCHING	

x_0 | x_1 | x_2 | x_3 → x_0 | x_1 | $(x_2 - x_3)$

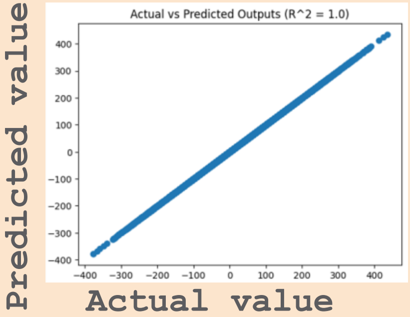
Testing Symmetries on the Mystery

Method	Output
LINEAR REG.	✗
POLYNOMIAL REG. #1	✗
SYMMETRY SEARCHING	



Testing Symmetries on the Mystery

Method	Output
LINEAR REG.	❌
POLYNOMIAL REG. #1	❌
SYMMETRY SEARCHING	x_0 | $x_1(x_2 - x_3)$
POLYNOMIAL REG. #2	$v_f^2 = v_0^2 + 2a(x-x_0)$



Bonus Features

x_0	x_1	x_0x_1	x_0^2	x_1^2
2	3	6	4	9
3	5	15	9	25
4	1	4	16	1
5	2	10	25	4

Cross Terms

Include products of different variables

Ex: $\boldsymbol{U_g} = \boldsymbol{m} \times \boldsymbol{g} \times \boldsymbol{h}$

$$y_f = y_0 - 4.9t^2$$

$$I = I_0 e^{-Lt/R}$$

$$U_e = \frac{1}{2}k(x_0 - x)^2$$

$$x = x_{\max} \cos(\omega t + \phi)$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

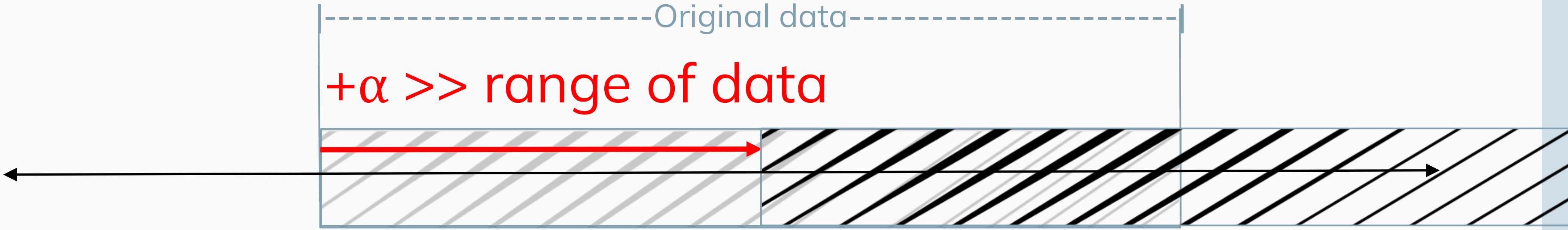
Functional Features

Apply a set of functions to all independent variables

x_0	$\sin(x_0)$	$\exp(x_0)$
$\pi/2$	1	$e^{\pi/2}$
π	0	e^{π}
$3\pi/2$	-1	$e^{3\pi/2}$

Practical Considerations

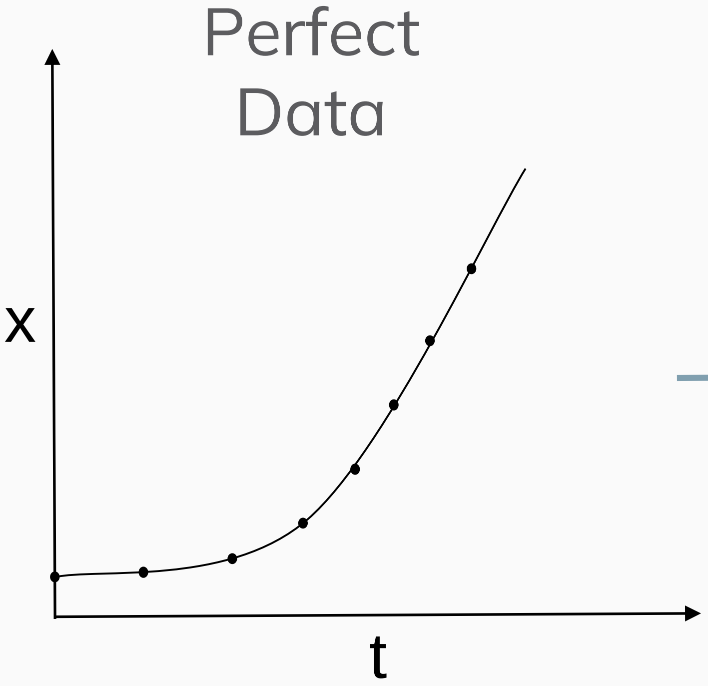
Neural Networks Aren't Perfect

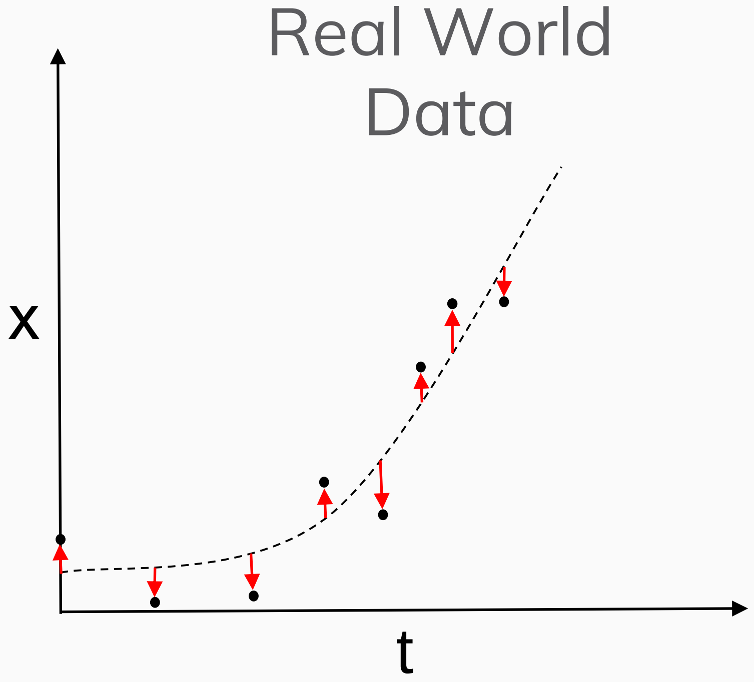


Our Approach?
Adaptive Transformation Deltas

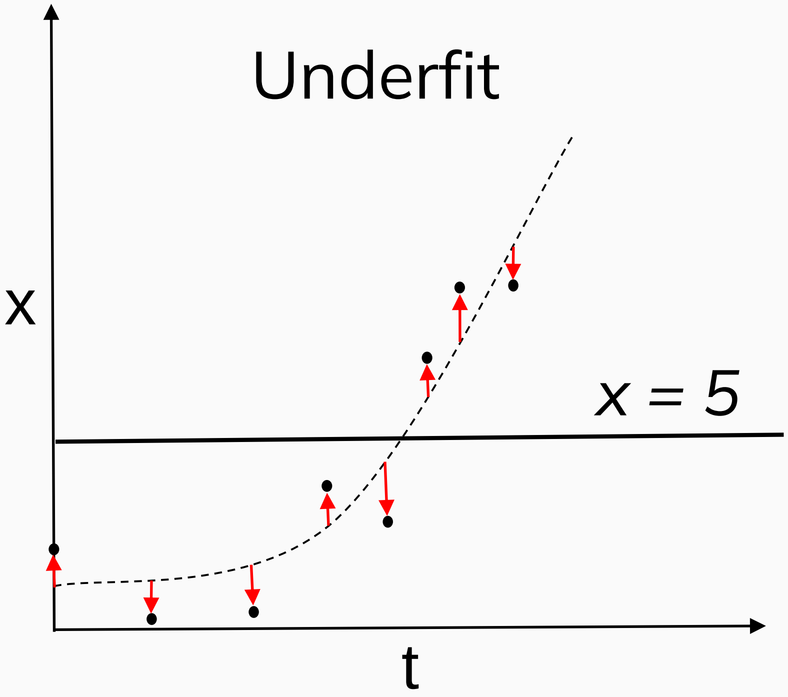
Real Data is Noisy

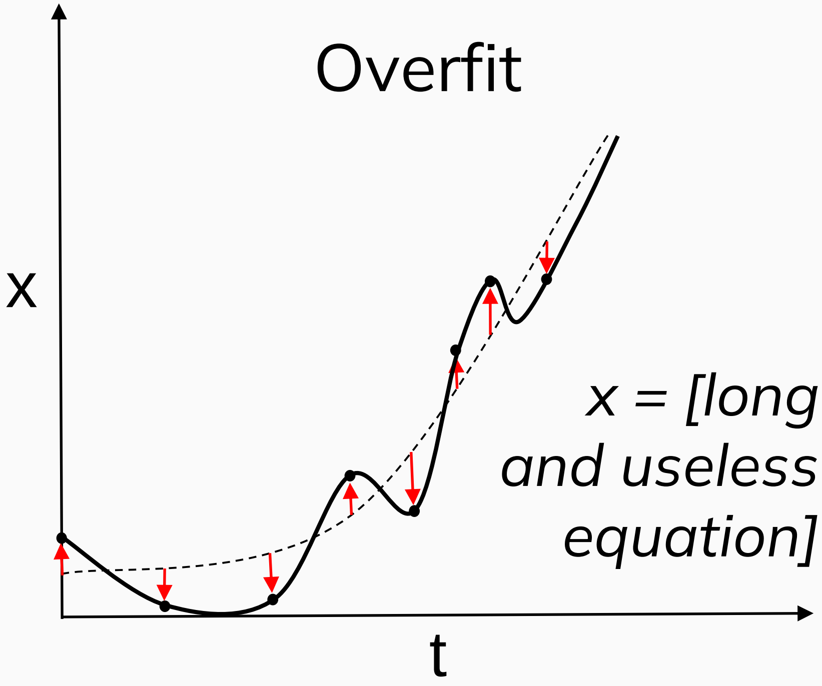
$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$





$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$





Conclusion

- Replicated and expanded on parts of “AI Feynman” from scratch
- Developed a single workflow for **discovering mystery functions**
- AI has potential to advance science
- Embedding **symmetries** is important
- The algorithm may be expanded to deal with more types of functions and noise in the future

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